Optimizing of reinforcing steel for the design of concrete columns

Bachelor thesis

To obtain the degree of Civil Engineer

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ACTA DE OBTENCIÓN DE TÍTULO

UNIVERSIDAD DE
GUANAJUATO

En la ciudad de Guanajuato, Gto. a las 10:00 (diez horas y cero minutos) del día 25 de julio del año 2019, se reunieron en el (la) Auditorio Ing. Salvador Yáñez Castro de la División de Ingenierías del campus Guanajuato de la Universidad de Guanajuato, los señores Maestro en Ciencias Humberto Esqueda Oliva, Doctor en Ingeniería Adrián David García Soto, Maestro en Ingeniería Francisco José Luna Rodríguez designados para verificar el examen de titulación del señor **LUIS FERNANDO VERDUZCO MARTÍNEZ**, concedido por acuerdo de la Rectoría General a efecto de obtener el título de **INGENIERO CIVIL**.

Instalado el jurado bajo la Presidencia del señor Maestro en Ciencias Humberto Esqueda Oliva y fungiendo como Secretario el señor Doctor en Ingeniería Adrián David García Soto, dio principio el examen e interrogaron sucesivamente los tres sinodales sobre diversas materias y aspectos del trabajo de titulación y, terminado el interrogatorio, se procedió a verificar la votación secreta, habiendo resultado el señor **LUIS FERNANDO VERDUZCO MARTÍNEZ**.

Aprobado

Para constancia se levanta la presente acta por triplicado que firman los señores sinodales.

Presidente

M. en C. Humberto Esqueda Oliva

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El suscrito, ***Maestro Francisco Javier Pérez Arredondo, Director de Administración Escolar****, hace constar que la presente acta constituye formalmente la realización del examen de obtención* **de Título de INGENIERO *CIVIL,*** *en los términos del Articulo 68 del Estatuto Académico de la Universidad de Guanajuato.*

SYNOD:

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Synod 2 M.SC. HUMBERTO ESQUEDA OLIVA

Synod 3 ENG.D. ADRIÁN D. GARCÍA SOTO

Dedication

La preocupación por el hombre y su destino siempre debe ser el interés primordial de todo esfuerzo técnico. Nunca olvides esto entre tus diagramas y ecuaciones.

...Albert Einstein

Regards and thanks

I take advantage hereby to do mention of the people who make this possible:

To my parents for their unconditional support.

To my aunt Lupita Navarrete, for having been my friend and an ethical mentor.

To my uncle Alfonso Pérez Áves for all his teaching and lessons in civil engineering.

To my aunt Maguín for having helped me out when I needed her to pursue some of my goals.

MANY THANKS

How come I'm here...

Everything began with my return from my first exchange program in the US, I had an enormous thirst to learn, to overcome myself and take advantage of as many opportunities I'd face to keep on growing personally and professionally.

Over that very summer of 2015 I heard of the 3th VHC (Verano de Herramientas Computacionales), and although I arrived a little bit late to the courses, I was still attending for the rest of the days left, and so I could realize at last, after my previous courses of Numerical Methods and Programming, of how much I could stimulate my creativity and my learning outcomes doing my own projects with programming.

However, I didn't practically nothing related with coding during the whole next semester, with the exception of some simple calculating excel sheets. It was until the course of "Concrete I" with Professor Dr. Alejandro Hernández Martínez and in my Social University Service with Profesor Eng. Francisco José Luna in the same year where I started doing more elaborated calculating excel sheets on my own, just to improve my learning outcomes with the subject, following the standard statement of *"Learning to code, coding to learn"*, and was thus, that for the final course project that I wrote my first macro in excel.

This way, I give thanks primarily to my Thesis' director and Professor of the course "Concrete I", ***Dr. Alejandro Hernández Martínez****, for having trusted me this theme, as he was the one who initially proposed it to me indeed, just after having concluded his course on the summer of 2016, this is, pretty early to begin a bachelor thesis along the whole program. He realized how much I'd enjoy making programs, and because of that I could keep on exploding my skills.*

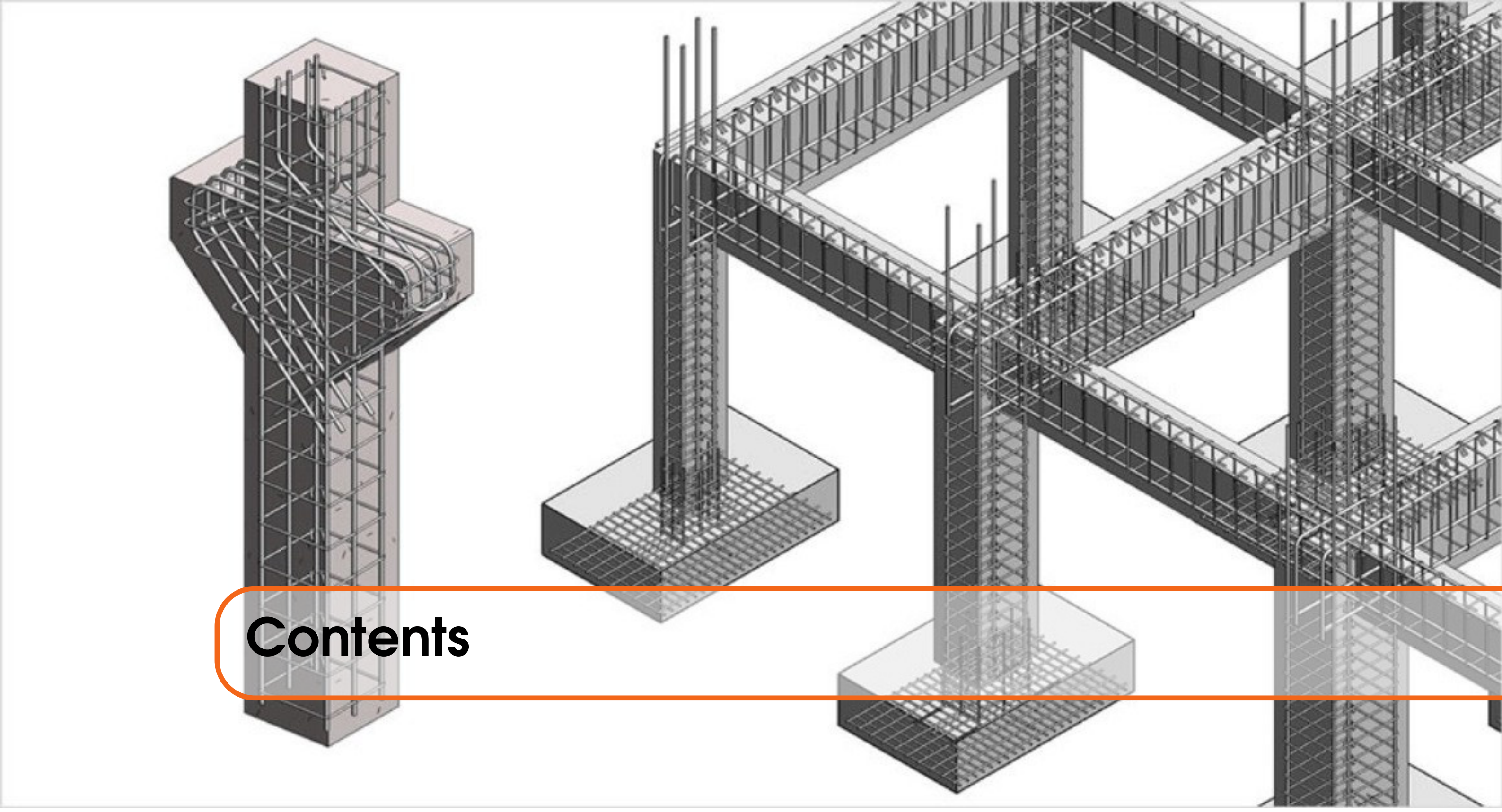
I want to thank as well Professor Eng. Francisco Luna for his lesson on Structural Analysis during his Social University Service program, in which I could take a lot bibliographical references to keep on learning programming along with some techniques for its application with structures.

From that point on, I started to work on this project; and again, thanks to my second exchange academic program (this time in Sweden), where I took some amazing courses in which I couldn't have challenged my programming skills more, when I learnt to create more complex algorithms. Hence, on my return from this program I was able to enter into the "Aula CIMNE-UG", where I got the opportunity to give it a larger sense to this beautiful art into civil engineering. Thus, I make a pause to thank Professor ***"Eng. Humberto Esqueda Oliva"****, an eminence in the area of programming and research, for having given me that honor.*

Once an Aula CIMNE member, I could learn a lot too of organization, entrepreneurship and team work, cooperating with the rest of the crew of such group, mainly ***Guillermo Yáñez y Miguel Angel Ríos Delgado****, true craftsman on engineering, who were pillars of motivation to keep on learning about computing.*

Without more, I hope this project I present hereby, may be a base of further research.

Luis F. Verduzco
Guanajuato, Guanjuato 2019



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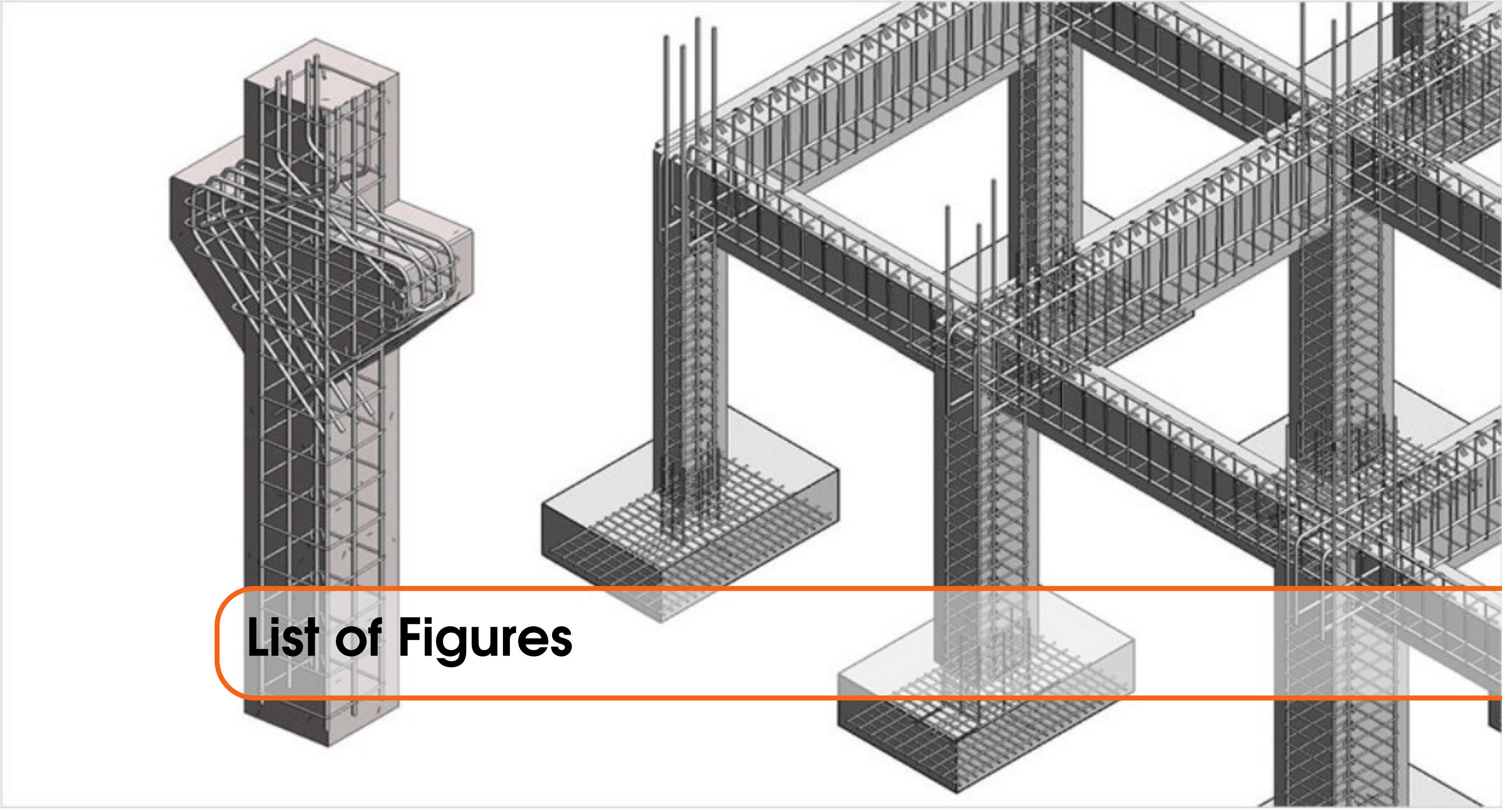

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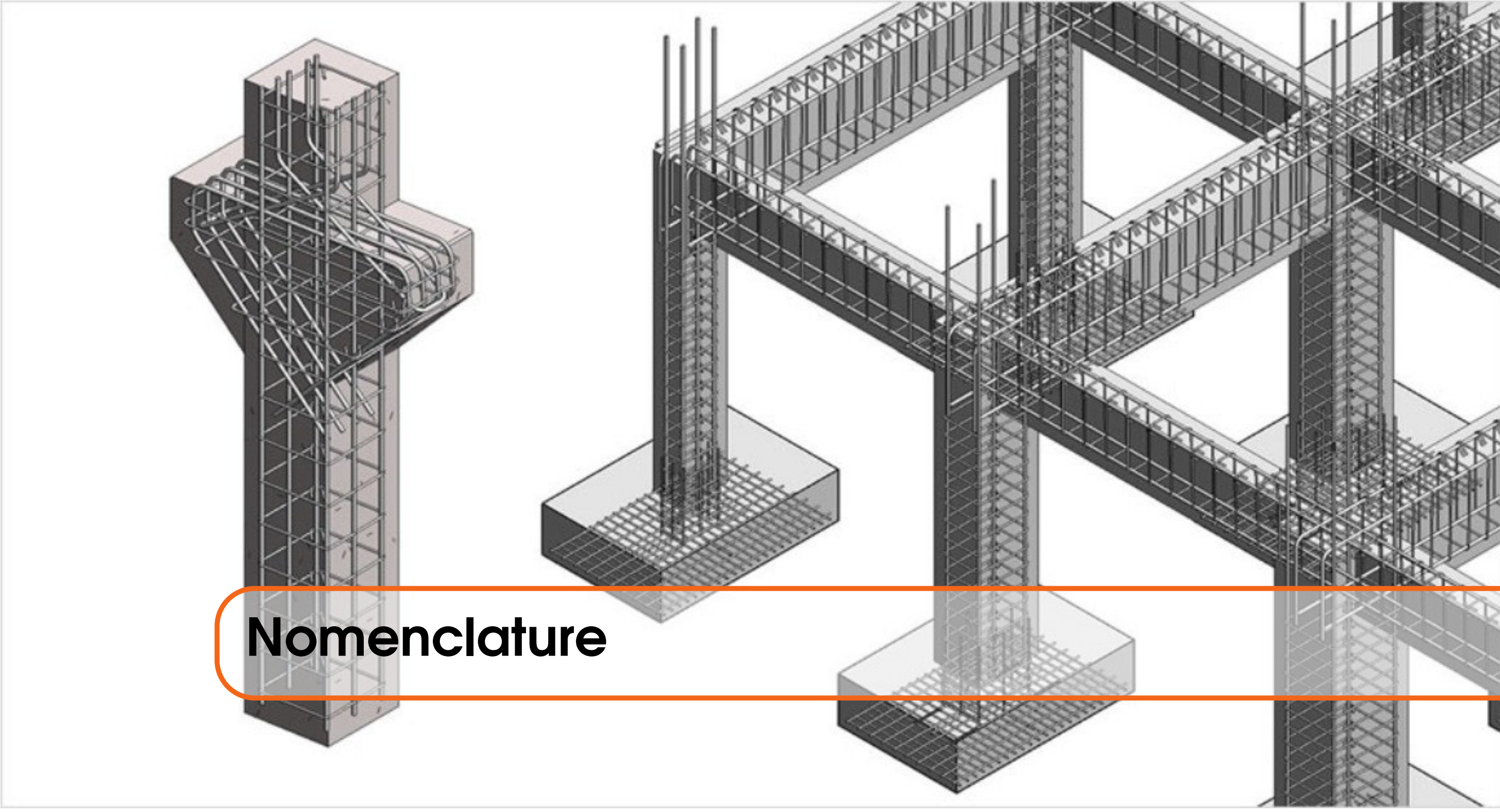
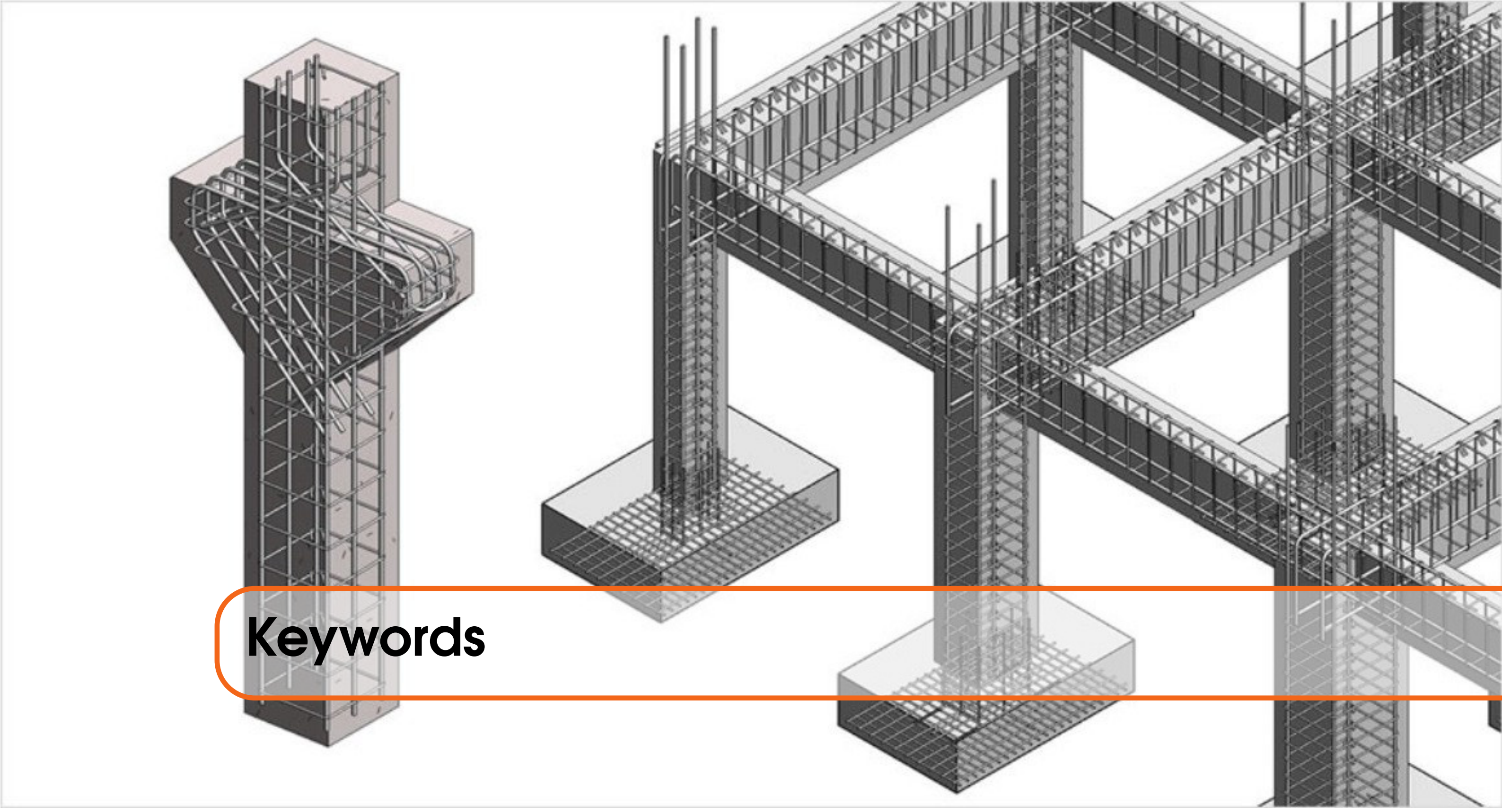


Nomenclature

Here below, the formal definitions of the most relevant terms and variables used in this project are presented.

Term	Definition	Units
P_{oc}	Resistance to compression stress of a reinforced concrete element	Ton/Kg
P_{ot}	Resistance to tension stress of a reinforced concrete element	Ton/Kg
fy	Creep strength of steel reinforcing bars of grade 42	Kg/cm^2
$f'c$	Resistance to simple compression stress of a concrete prism	Kg/cm^2
A_s	Reinforcing steel area over a concrete element cross section	cm^2
c	Neutral stress axis position along a structural element cross section	cm
ε_u	Plane strain of concrete to reach creep limit	—-
β_1	Resistance reduction factor of simple compression stress	—-

Term	Definition	Units
P_R	Maximum normal load over the element cross section acting at eccentricities e_y y e_x.	Ton/Kg
a	Reduced position of neutral axis for the transformation of real stresses to the equivalent rectangular block of compression stress	cm
b	Width of a rectangular concrete column cross section	cm
C	Net resistant force to compression of concrete at a certain position of the neutral axis	Ton/Kg
P_{rx}	Normal max load acting at a eccentricity e_y	Ton/Kg
P_{ry}	Normal max load acting at a eccentricity e_x	Ton/Kg
h	Height of rectangular concrete element cross section	cm
t	Width of idealized steel profile embedded in a concrete element	cm
d_1	Distance from the uppermost cross section edge of a rectangular concrete element to the middle of the steel profile upper horizontal width	cm
d_2	Distance from the uppermost edge of the rectangular concrete element cross section to the middle of the steel profile lower horizontal width	cm
rec	Concrete covering width for reinforcing steel against weathering	cm
M_{ux}	Bending resistant moment over the reinforced concrete element cross section respect to the axis x	$Kg \cdot cm/Ton \cdot m$
M_{uy}	Bending resistant moment over the reinforced concrete element cross section respect to the axis y	$Kg \cdot cm/Ton \cdot m$
M_{ux}	Acting bending moment over the reinforced concrete element cross section respect to the axis x	$Kg \cdot cm/Ton \cdot m$
M_{uy}	Acting bending moment over the reinforced concrete element cross section respect to the axis y	$Kg \cdot cm/Ton \cdot m$
E_{ac}	Modulus of elasticity of the reinforcing steel grade 42	Kg/cm^2
F_a	Resistant force of the horizontal upper part of the idealized steel profile	Ton/Kg
F_b	Resistant force of the horizontal lower part of the idealized steel profile	Ton/Kg
d_{ma}	Diameter of circular column	cm



Keywords

Efficiency:	Ratio between the acting load and the resistance force of the column. Must be always less than 100%. Efficiencies between 80% and 90% are the most commonly required.
Steel optimization:	Numerical calculation through which the most economical and structural arrangement of steel bars is found.

Part 1

1. Introduction

It is often required in projects of design and structural analysis for concrete elements of numerical calculations and processes that may evoke several iterations in order to get a structural as well as a economical efficient result; with the usage of numerical methods and algorithms, solutions might be encountered more rapidly.

The usage of numerical methods and programming has become a totally necessary tool for the automation and optimization of precesses regarding time and resources for the design of civil infrastructure. It is, thus, essential, as an engineer, in any area that might be implied, to have knowledge as well as skills in computing and programming, either in one or various platforms and programming languages interfaces.

The present work is oriented precisely to use programming and numerical methods as an example of the application of such for the design and analysis of civil infrastructure. In this case, the necessary reinforcing steel for concrete columns will be determined to withstand certain load conditions.

In common practice, the determination of this such necessary reinforcing steel in concrete columns is realized through empirical approximations based on mere experiences; the design process becomes thus, overwhelming and tiring.

Hereby, the design of software will be exposed, whose main function is to determine the reinforcing steel bars arrangement for a concrete column that may be the most structurally and economically efficient undergoing bending-compression. The designer, therefore, will have to define solely the geometrical dimensions of the concrete elements cross sections in function of the structural and/or spacial requirements. These elements could be short and/or long columns, beams, pillars, or even foundation dies.

The program has the capacity to analyse concrete structural elements of both rectangular and circular cross section geometries. Due to constructive means, these two geometries are most usually required, because of their uniformity forms.

As part of the results of this program, proposals as well as recommendations of steel reinforcing bars arrangements are obtained, that is, not only the options of number of reinforcing bars are encountered with their respective diameter, but also the arrangement and position over the cross sections of each bar, indicating as well the critical structural efficiency for each arrangement. The program selects either the most structurally efficient arrangement or the cheapest (according to the constructor preferences) for each column, to then calculate the volumes of the required materials (steel, concrete and form-work) for the integration of the construction project quote. Going from the project design stage to the work budget in a matter of seconds. Several commercial software are not able to calculate this, only results of analysis with the initially given cross section geometries and arrangement of steel bars are obtained.

The applications and scope in general for this project and focus of analysis are of great range in the design of concrete structures, due to its possibility of adaptation to any geometry.

1.1 Objetive

To design a program with the capacity to determine all the possible arrangements of reinforcing steel bars over any number of concrete columns, select the most efficient/cheap option for each column, and integrate the whole project work budget.

2. Background and generalities

2.1 Classification of concrete columns

[1]

Reinforced concrete columns may be classified as short columns, middle columns and long columns. It is essential to take this on account in order to understand better how the program functions, because the mere analysis for each type of column precedes from its failure mechanism.

Short columns: Their bearing load capacity it's governed by their cross section dimensions and by the material by which its made.

Middle columns: Their failure it's due to a mixture of crushing and buckling.

Long columns: Their bearing axial load capacity it's reduced by the secondary bending moments as result from the same bending deformation of the column.

2.2 Types of concrete columns

[2]

The type of column depends on the shape of its cross section, as well as reinforcing type. There are concrete columns with longitudinal reinforcing steel bars, either with spiral lateral restriction or with closed stirrups.

Columns with closed stirrups have generally a rectangular cross section, while on the other hand, columns with spiral lateral restriction are used to be of circular cross section,

[1] Jack C. McCormac, Rusell H. Brown, *"Diseño de concreto reforzado"*, 14va edición, Alfaomega(2015), p.257.
[2] Jack C. McCormac, Rusell H. Brown, *"Diseño de concreto reforzado"*, 14th edition, Alfaomega (2015), p. 258.

although octagonal cross sectioned columns might be prefabricated as well, among other geometries. Spiral reinforcement in comparison with stirrups increase the resistance on a major rate due the effect of confinement , even though in some cases, increasing also construction costs.

2.3 General basic concepts

It is absolutely necessary to comprehend the following hypothesis for reinforced concrete design, as they are where the employed factors for the analysis come from.

2.3.1 Axial load

[3]

Let us say, a simple concrete prism of rate height width equal to two is subjected to an axial compression load, then the maximum load will take place at a unit strain of 0.002. Arbitrarily, a 100% of resistance is considered for a prism of a slenderness ratio of two.

For slenderness ratios more than two, the resistance decreases to reach a 85 percent, approximately. Hence, the resistance of a simple concrete element subjected axial compression may be estimated as the product of the 85 % of the measured stress for the cylinder with f'_c tested under the same conditions, multiplied by the element cross section area. This reduction factor of 0.85 is just an average of different results taken from tests of concrete prisms cast vertically.

When reinforcing steel bars are added to a simple concrete specimen as well as the necessary transversal reinforcement to keep the longitudinal bars on place during casting, then the maximum load will be obtained at a unit strain fo 0.0021. Failure, on the other hand, will take place a unit strain between 0.003 and 0.004, if the test is of short duration.

The additional resistance over a simple concrete prism due to the addition of longitudinal reinforcing steel in compression may be estimated as the product of the net steel area by the creep stress factor f_y

Therefore, the resistance or maximum compression load that a reinforced concrete prism might withstand will be given by the following expression:

$$P_{oc} = (0.85)f'_c(A_c - A_s) + f_y(A_s) \tag{2.1}$$

It is worth stressing that the contribution of the steel area over the compression concrete resistance is not being considered (first term of the ***Equation 2.1***), as this might have great influence over the calculation results.

On the other hand, the maximum bearable tension load for the concrete element may be given by:

[3]González Cuevas, Fco. Robles Fernández, *"Aspectos básicos del Concreto Reforzado"*, 4ª edición, Limusa (2005). Capítulo 5, p. 83-87.

$$P_{ot} = (f_y(A_s)) \tag{2.2}$$

In which only the steel intervenes, as the concrete cracks and does not contributes to the resistance.

2.3.2 Bending

General hypothesis for bending design

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a. Deformation of concrete at a given point over the element cross section is proportional to the distance of such point with respect to the neutral axis.
b. Reinforcing steel has a perfect elastic plastic behaviour on compression and identical to tension.
b.1 Unit creep strain is considered as 0.0021
c. Real concrete compression stress distribution over a given cross section may be transformed to a equivalent stress block. **Figure 2.1**
d. There is not relative sliding between the reinforcing steel and the surrendering concrete. It is supposed that the unit strain is the same on the reinforcing steel as well as on the concrete, considering the same position level over the cross section.
e. Concrete does not bear up tension stresses.
f. The element reaches its resistance at maximum concrete unit strain of 0.003.

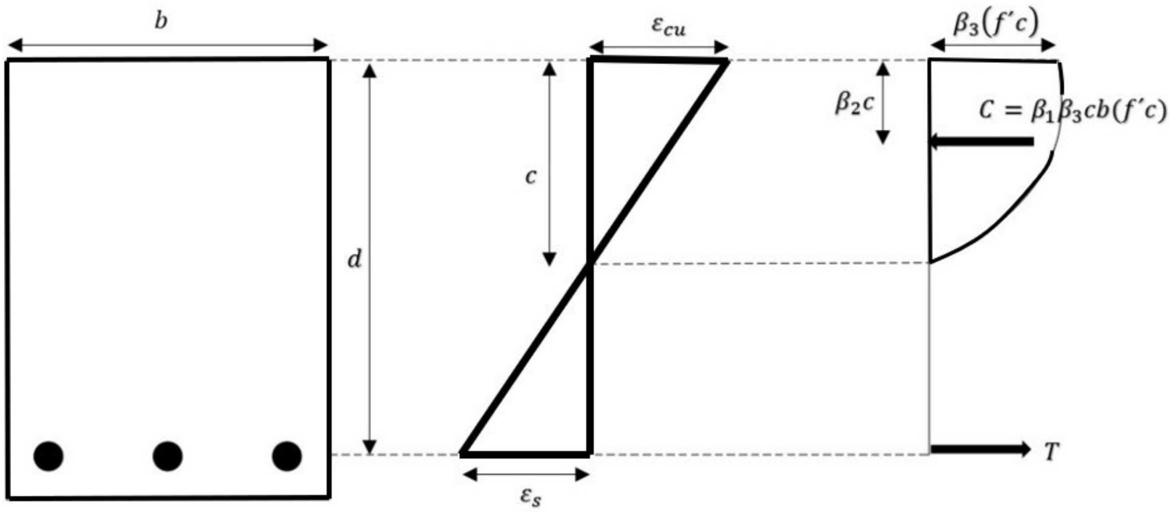

Figure 2.1: *Equivalent compression stress block for concrete (Adapted from: [2])*

[4] González Cuevas, Fco. Robles Fernández, *"Aspectos básicos del Concreto Reforzado"*, 4ª edición, Limusa (2005). Capítulo 5, p. 83-87.

Determination of pure bending resistance

In order to calculate the pure bending resistance of a concrete element there must be an equilibrium of forces with respect to the centroid axis of the element, pretending to obtain the location of the neutral axis c [5], and from there on, to determine the equivalent stress block for concrete, the forces produced by the longitudinal reinforcing steel as well as their acting position over the cross section, and finally the bending moments produced by such forces around the cross section centroid axis of the element. **Figure 2.2**

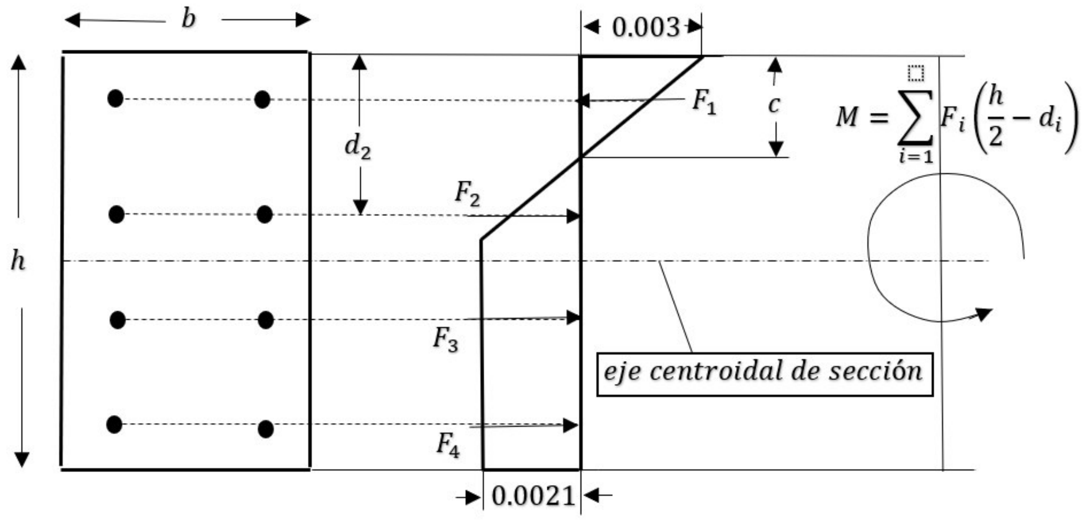

Figure 2.2: *Stress distribution over a concrete cross section subjected to bending. (Own made drawing).*

The bending moments sign convention will be:

(+) counter clockwise

(-) clockwise

[5]Transversal cross section axis over which space there are not stresses, being this such linear space the transition between tension and compression over the given cross section.

2.3.3 Bending-compression

General considerations for the analysis

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a. The analysis is addressed through proposals of cross section geometries that will remain constant, being the steel reinforcing area the only variant.
b. An element might reach its resistance under several axial-bending load combinations. This combinations go from an axial maximum load (P_{oc}) with no bending moment to a maximum bending moment (M_o) with no axial force.
c. The geometrical space that depicts all possible combinations of axial-bending loads with which an element may reach its resistance is represented by a *"Interaction diagram"* **Figure 2.3**.
d. When increasing the external load, the axial and bending loads increase as well with the same proportion, therefore the load history is depicted by a straight line going from the origin, with a slope equal to the quotient $P/M = 1/e$. **Figure 2.3**.
e. There are only two main failure types for concrete elements undergoing bending- compression: *compression failure and tension failure:*

    1. *Compression failure*: It's produced by crashing of concrete. The most compressed steel area creeps in compression, while the opposite side does not creep in tension.
    2. *Tension failure*: It's produced when the steel area in tension at one side creeps before crushing of concrete at the opposite side occurs.

f. The interaction diagram of an element may be obtained from the described hypothesis for the calculated resistance of elements subjected to pure bending, considering now that the sum of resistant forces must be equal to the applied load P.

NOTE: *The previous considerations apply for any geometry.*

Bresler formula

Bresler developed a very simple expression for rectangular columns to calculate the maximum values of the compression load acting at eccentricities e_x and e_y on rectangular cross sections with symetrical reinforcement. ***Equation 2.3***. Such solution arose before the necessity of avoiding certain overwhelming calculations to determine an *interaction surface*[7], reducing the problem to a more simple combination of solutions; two bending-compression loads in one symmetry plane and one axial compression force.

[6] González Cuevas, Fco. Robles Fernández, *"Aspectos básicos del Concreto Reforzado"*, 4ª edición, Limusa (2005). Capítulo 6, p. 127-155.

[7] It's the geometrical space of an element cross section in which all the values of resistance bending- forces in every existing plane are depicted

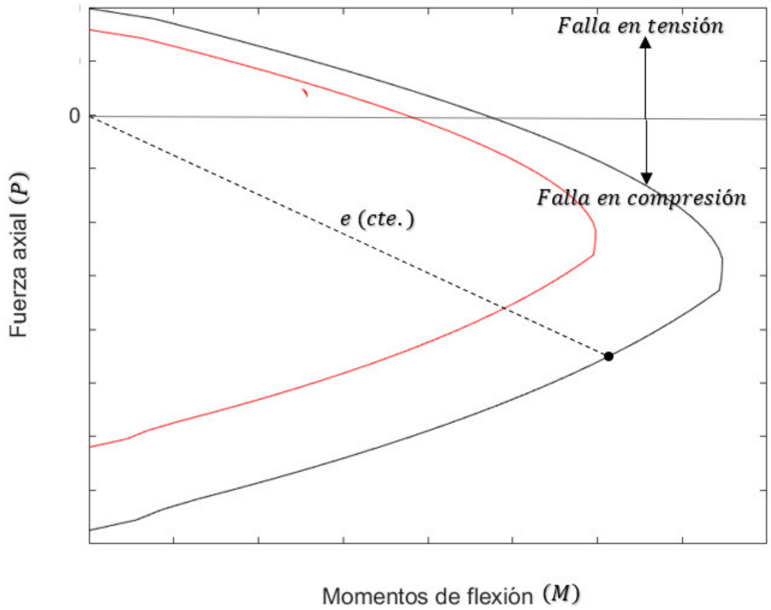

Figure 2.3: *Typical interaction diagram (**Own made drawing.**)*

$$\frac{1}{P_R} = \frac{1}{P_{rx}} + \frac{1}{P_{ry}} - \frac{1}{P_{oc}} \quad (2.3)$$

Where:

P_R=Normal resistant load over the element cross section acting at eccentricities e_x and e_y.
P_{rx}=Normal resistant load acting at an eccentricity of e_y.
P_{ry}=Normal resistant load acting at an eccentricity of e_x.

The ***Equation [2.3]*** verifies all the available tests inside the 20% of approximation, and represents a whole family of planes converging to the interaction surface points. **Figure 2.4**.

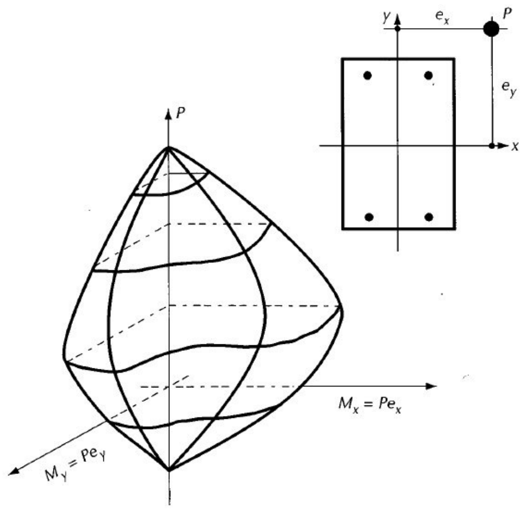

Figure 2.4: *Typical interaction surface* ***Own made drawing:[2]***

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Another way of analysing columns subjected to biaxial bending- compression ***Equation [2.4]*** which is the one from which another simplified equation arises to calculate the efficiency of an element when the ratio $P_R/P_{oc} < 0.1$. ***Equation [2.9]*** (*and is the one presented in the norms NTC-2017* ***[3]****)* used for infrastructure design, and to which reference is given after this section.

$$(\frac{P_u - P_{nb}}{P_{ot} - P_{nb}}) + (\frac{M_{ux}}{M_{nbx}}^{1.5}) + (\frac{M_{uy}}{M_{nby}}^{1.5}) = 1.0 \tag{2.4}$$

Where:

P_u=Axial nominal load applied

P_{nb}=Nominal resistance against axial load applied at a balanced condition.

M_{nbx} and M_{nby}=Nominal resistant bending moments at the balanced condition around axis X and Y respectively.

M_{ux} y M_{uy}=Nominal bending moments applied around axis X and Y respectively.

2.4 Normative

General hypothesis for design of concrete were presented in previous sections because those are the hypothesis from which construction norms are based. In order to develop and complete this project, reference to the norms **Normas Técnicas complementarias para el diseño y construcción de estructuras de concreto del Reglamento de Construcciones de la Ciudad de México (NTC-2017)** was given.

2.4.1 Axial load

In the NTC-2017 it is considered necessary to make a modification to the value f'_c through load factors F_c and Resistance factor F_R[8]. At the section given for concrete it is specified that for the calculation of resistance a reduced resistance to compression should be used, denominated $f''c$. Whose value is:

$$f''c = 0.85(f'c) \tag{2.5}$$

Hence, the equation ***Equation [2.1]*** is transformed to:

$$P_{oc} = F_R((f''c)(A_c - A_s) + f_y(A_s)) \tag{2.6}$$

And the equation ***Equation [2.2]*** to:

$$P_{ot} = F_R(f_y)(A_s) \tag{2.7}$$

For a better interpretation of this such mentioned factors, reference may be made from the **Figure 2.5**, which is equivalent stresses block of transformation from the hypothesis of the NTC-2017.

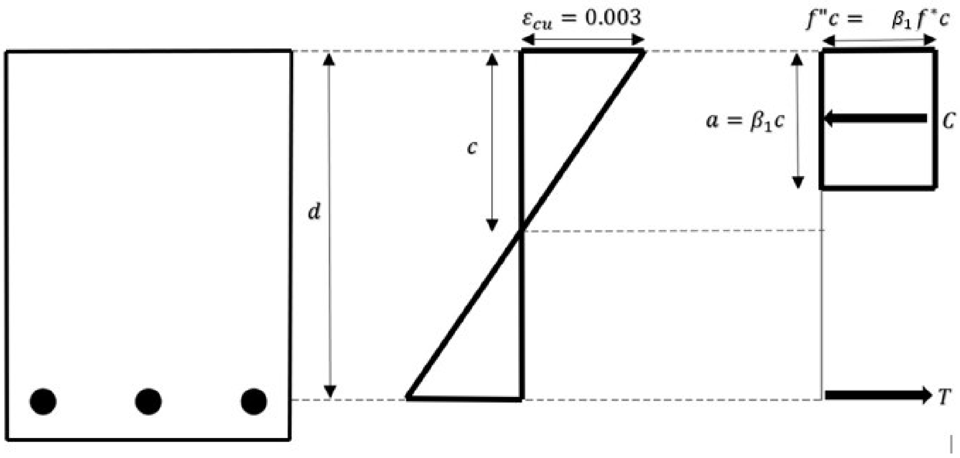

Figure 2.5: *NTC-2017 hypothesis for linear strain distribution on the zone subjected to compression.* ***Adapted from: [3]***

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[8]Reduction resistance factor with value 0.8 for columns subjected to axial load ***[3]***

Where:

$$C = ab(f''_c)$$
$$0.65 <= (\beta_1 = 1.05 - \frac{f'_c}{1400}) <= 0.85$$

2.4.2 Bending and compression in two axial directions

[9]

The following expression (arisen from the **Bresler** formula) will be used for the calculation of the maximum acting load over the cross section at a certain eccentricity.

$$P_R = \frac{1}{\frac{1}{P_{rx}} + \frac{1}{P_{ry}} - \frac{1}{P_{oc}}} \tag{2.8}$$

For values of $\frac{P_R}{P_{oc}} < 0.1$, the following expression will be used instead:

$$\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} <= 1.0 \tag{2.9}$$

Where:

M_{ux} and M_{uy} are the acting bending moments around the axis X, Y, respectively.

M_{rx} y M_{ry} are the design resistant bending moments around the the same axis.

[9] González Cuevas, Fco. Robles Fernández, *"Aspectos básicos del Concreto Reforzado"*, 4ª edición, Limusa (2005). Capítulo 6, p. 148.

Part 2

3. Analysis approach

The demonstration of the analysis approach will be presented for both (*rectangular and circular*) cross section geometries, in chronological order for the neutral axis position along the height of the section, from -∞ to ∞ .

3.1 Rectangular columns

Making use of the previously design hypothesis an idealized reinforcing steel shape has been made up, as shown next: **Figure 3.1**

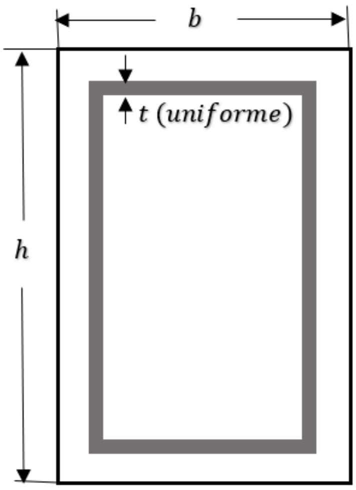

Figure 3.1: *Rectangular idealized cross section of reinforced concrete*

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Where:

t = idelized steel cross section profile width as constant on its perimeter.

3.1.1 Analysis cases for calculation of steel reinforcement resistance

According to linear strain distribution and elastic-plastic behaviour of steel, five different analysis cases might derived with respect all the different neutral axis positions for the calculus of the interaction diagram, adapting to all the different geometries that might be presented from the strain state over the element cross section. All these strain geometry configurations may be transformed to stresses and consequently to punctual forces **Figure 3.2**, with which the acting bending moment respect to the neutral axis might be calculated as well.

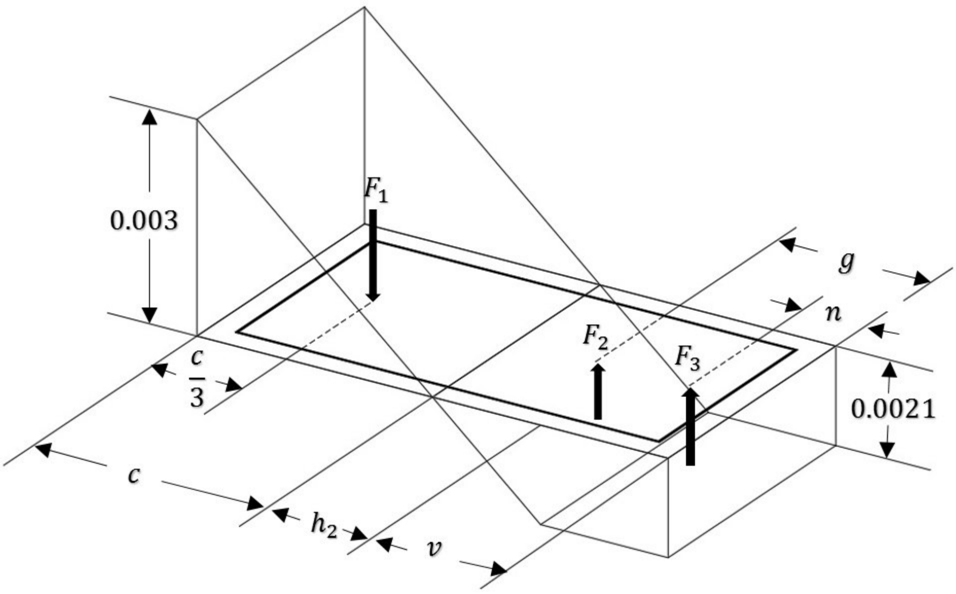


Figure 3.2: *Strain distribution block geometry for a certain position of neutral axis along the element cross section.*

Methodology

The following methodology for the analysis of bending moments and forces is presented in a general manner, with which the developed formulas and equations might be conceptualized and segmented.

For the resultant resistant forces:

For each block (i) under certain strain state:

- To determine the geometrical configuration length (L) along the section height.
- To determined the effective area under stress $2tL$
- To determine the resultant force governed by the given geometry configuration of stresses

$$F_i = \begin{cases} 2E_{ac}tL\varepsilon & \text{si } \varepsilon <= 0.0021 \\ 2f_y tL & \text{si } \varepsilon > 0.0021 \end{cases} \tag{3.1}$$

Where:

$$E_{ac} = \frac{f_y}{\varepsilon_y} = \frac{\frac{kg}{cm^2}}{0.0021}$$
$$E_{ac} = 2e10\frac{kg}{cm^2}$$

- To sum al the resultant forces of each geometrical configuration of stresses to obtain the net resistance:

$$F_R = \sum_{i=1}^{i=no.bloques} F_i$$

For the resultant resistant bending moments:

For geometrical configuration (i) in a determined stress state:

- To determine the depth d_i of the geometrical configuration centroid over which the resultant force acts.
- To determine the distance fron that centroid point with respect to the cross section centroid axis $\frac{1}{2}h - d_i$
- To determine the equivalent resultant bending moment:

$$M_i = F_i(\tfrac{1}{2}h - d_i)$$

- To sum al the resultant bending moments produced by each geometrical stress configuration to obtain the net resistant bending moment:

$$M_R = \sum_{i=1}^{i=no.bloques} M_i$$

It's important to take on account that only the reinforcing steel is being considered so far. Hence, in order to calculate the net volume of the geometrical configuration (resultant forces), both lateral parts of the steel profile are analysed separately **Figure 3.3** from the upper and lower parts. Thus, the forces (F_a and F_b) ***Equation [3.2]*** and ***Equation [3.3]*** will take place for every analysis case, representing the net resistant forces acting at the centroid of this such upper and lower steel parts, taken as d_1 and d_2, respectively.

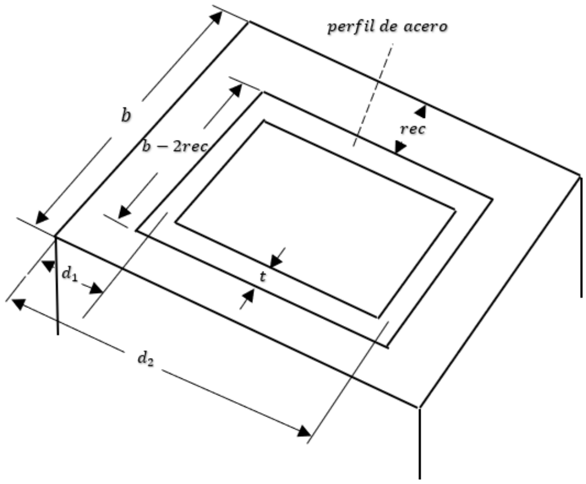


Figure 3.3: *Rectangular idealized section for reinforced concrete*

.

$$F_a = \varepsilon_a E_{ac} t (b - 2rec) \quad (3.2)$$

$$F_b = \varepsilon_b E_{ac} t (b - 2rec) \quad (3.3)$$

Note: *The sign convention will be:* ***Compression (-)*** *and* ***Tension (+)****.*

3.1.2 Case 1: One geometrical configuration of stresses in tension takes place

$$-\infty < c <= \frac{30}{51} d_1$$

See demonstration 1.1 in Annex 1 [p. 73]

There does not exist bending moments over the element cross section for this case, but only axial tension forces. **Figure 3.4**

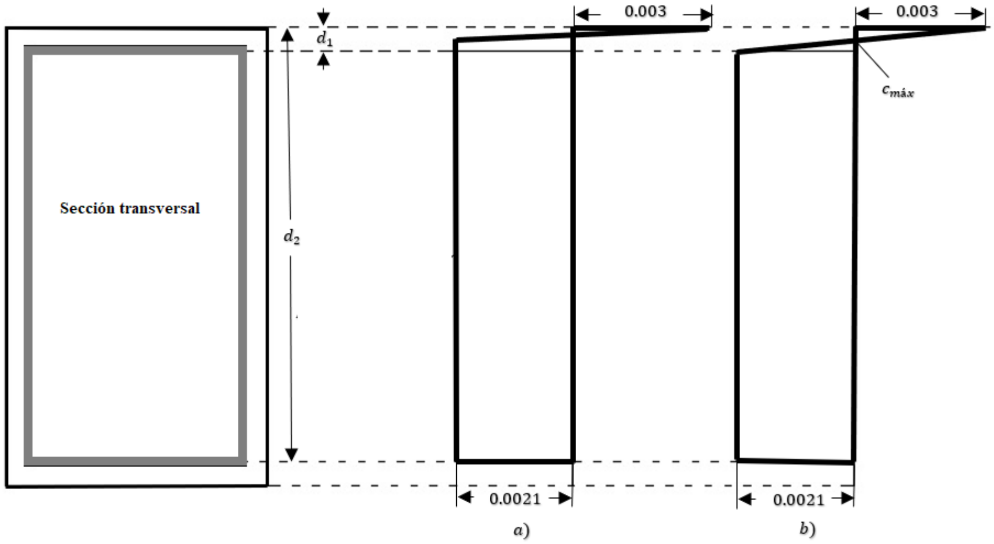


Figure 3.4: *Strain distribution diagram. Case 1. a) At some given position of c inside the allowed range, b) c is at superior boundary of the allowed range.* ***Own made drawing***. *See demonstration 1.1 in Annex 1 [p. 73]*

Where:

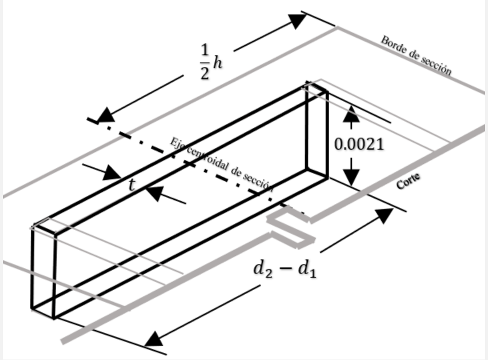
$$d_1 = rec + \frac{1}{2}(t)$$
$$d_2 = h - rec - \frac{1}{2}(t)$$

Thus, from equations ***Equation 3.2*** and ***Equation 3.3***:

$$F_a = 0.0021 E_{ac} t (b - 2rec) \tag{3.4}$$

$$F_b = 0.0021 E_{ac} t (b - 2rec) \tag{3.5}$$

$$F_1 = 2(t)(d_2 - d_1)E_{ac}(0.0021) \qquad (3.6)$$



$$M_1 = 0$$

Then, the force and net bending moment result as follows:

$$F_R = F_1 + F_a + F_b \qquad (3.7)$$

$$M_R = M_1 \qquad (3.8)$$

3.1.3 Case 2. Three geometrical configurations take place, two rectangular and one triangular, all of them in tension

Giving continuity to the previous case, for this case 2 the allowed range in which the position of c might be is:

$$\frac{30}{51}d_1 < c <= d_1$$

Let us make reference to visualize this range **Figure 3.5**.

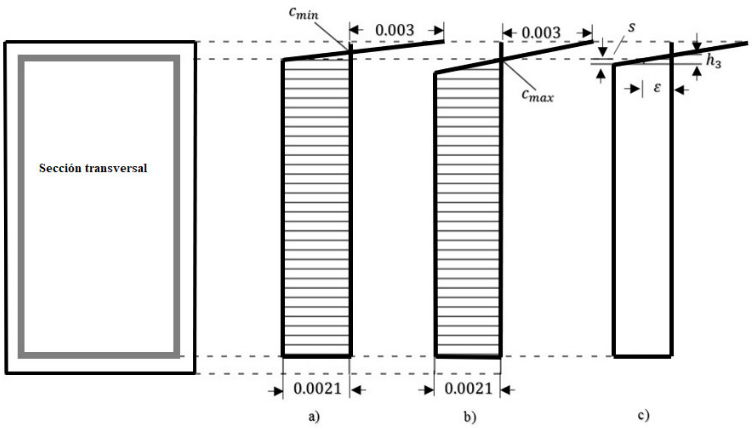


Figure 3.5: *a) Strain distribution diagram with c at the inferior boundary of its allowed range, b) Geometrical strain configuration diagram with c at the superior boundary of its allowed range, c) Geometrical strain configuration diagram at any position of c inside its allowed range.* ***(Own made drawing)****. See demonstration 2.1 in Annex 1 [p. 74]*

Thus, from equations ***Equation 3.2*** and ***Equation 3.3***:

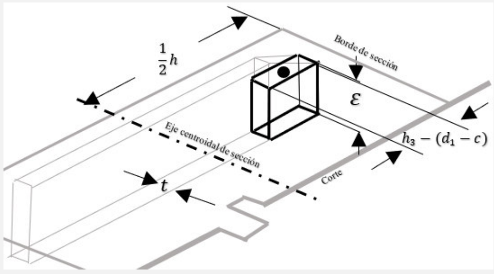
$$F_a = 0.0021 E_{ac} t (b - 2rec) \quad (3.9)$$

$$F_b = 0.0021 E_{ac} t (b - 2rec) \quad (3.10)$$

Now, with the previous defined variables and from **Figure 3.5**, the following equations may be deducted in order to calculate the resultant resistant force and bending moment for reinforcing steel.

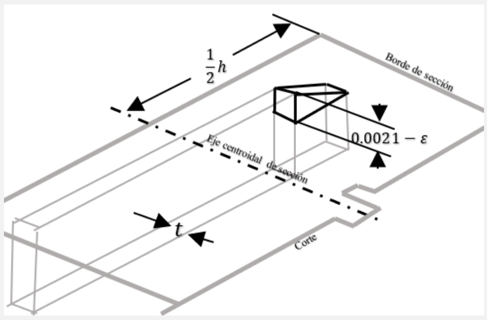
$$F_1 = 2(t)(\varepsilon)(h_3 - (d_1 - c))E_{ac} \quad (3.11)$$

$$M_1 = F_1(d_1 + \frac{1}{2}(h_3 - (d_1 - c) - \frac{1}{2}h) \quad (3.12)$$



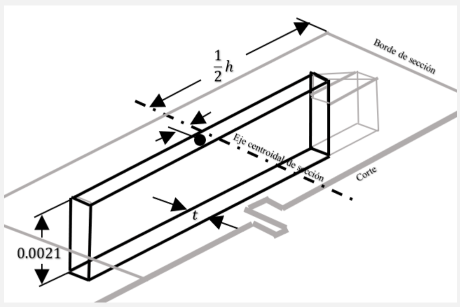
$$F_2 = 2(t)\frac{1}{2}(0.0021 - \varepsilon)(h_3 - (d_1 - c))E_{ac} \quad (3.13)$$

$$M_2 = F_2(d_1 + \frac{2}{3}(h_3 - (d_1 - c)) - \frac{1}{2}h) \quad (3.14)$$



$$F_3 = 2(t)(d_2 - h_3 - c)(0.0021)E_{ac} \quad (3.15)$$

$$M_3 = F_3(c + h_3 + \frac{1}{2}(d_2 - c - h_3) - \frac{1}{2}h) \quad (3.16)$$



Hence, the net resistante normal force and bending moment remain as follows:

$$F_R = F_1 + F_2 + F_3 + F_a + F_b \tag{3.17}$$

$$M_R = M_1 + M_2 + M_3 + F_a(d_1 - \frac{1}{2}h) + F_b(d_2 - \frac{1}{2}h) \tag{3.18}$$

3.1.4 Case 3; Three strain geometrical configurations take place, one triangular in compression, one triangular in tension and other rectangular in tension.

The allowed range of position for the neutral axis c is:

$$d_1 < c <= \frac{30}{9}d_1$$

Let us make reference to **Figure 3.6** to visualize this range.

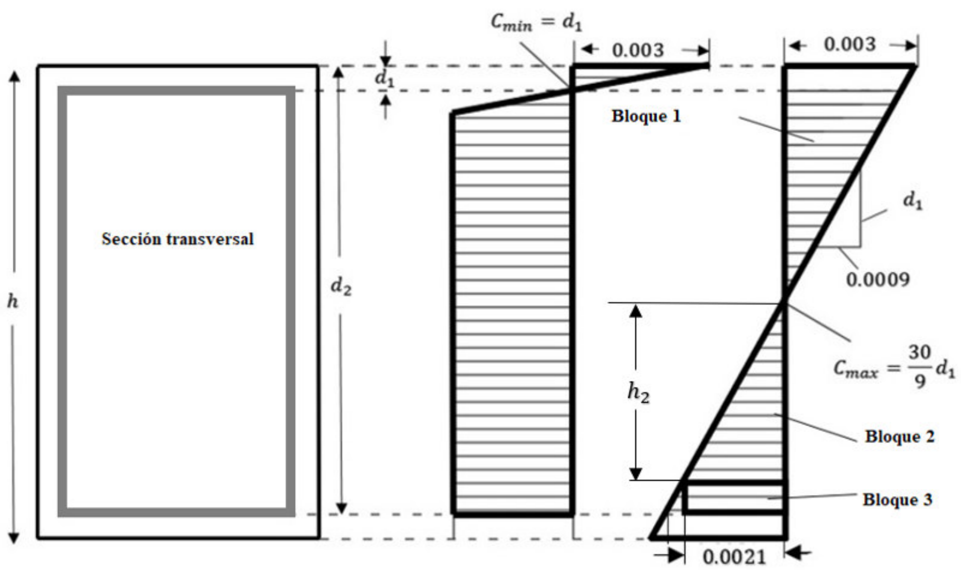


Figure 3.6: *a) Cross section, b) Geometrical strain configurations at the lower boundary of the allowed range for c, c) Geometrical strain configurations at the upper boundary of the allowed range for c* ***Own made drawing****. See demonstration 3.1 in Annex 1 [p. 74]*

Where:

$$h_2 = \frac{21}{30}c$$

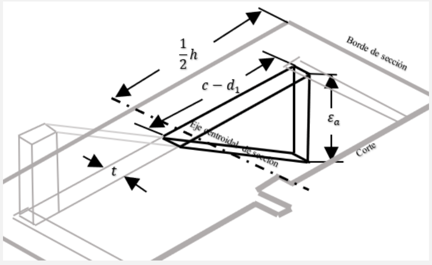
Thus, the following unit strain values are given in order to define equations ***Equation 3.2*** and ***Equation 3.3***:

$$\varepsilon_a = (1 - \tfrac{d_1}{c})(0.003)$$
$$\varepsilon_b = 0.0021$$

Hence, the following equations arise for the net resultant normal forces and bending moments:

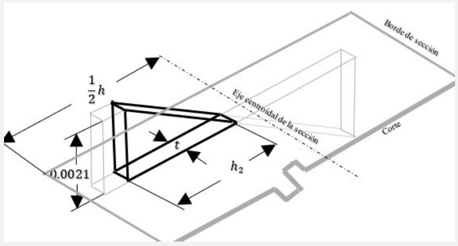
$$F_1 = -\frac{1}{2}(c - d_1)(\varepsilon_a)(2t)E_{ac} \qquad (3.19)$$

$$M_1 = F_1(d_1 + \frac{1}{3}(c - d_1) - \frac{1}{2}h) \qquad (3.20)$$



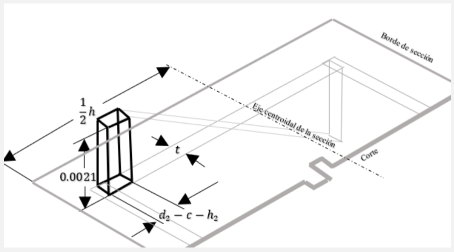
$$F_2 = \frac{1}{2}h_2(0.0021)(2t)E_{ac} \qquad (3.21)$$

$$M_2 = F_2(c + \frac{2}{3}h_2 - \frac{1}{2}h) \qquad (3.22)$$



$$F_3 = 0.0021(d_2 - c - h_2)(2t))E_{ac} \qquad (3.23)$$

$$M_3 = F_3((c + h_2) + \frac{1}{2}(d_2 - c - h_2) - \frac{1}{2}h) \qquad (3.24)$$



$$F_R = F_1 + F_2 + F_3 - F_a + F_b \qquad (3.25)$$

$$M_R = M_1 + M_2 + M_3 - F_a(d_1 - \frac{1}{2}h) + F_b(d_2 - \frac{1}{2}h) \qquad (3.26)$$

3.1.5 Special case. Two geometrical strain configurations take place, both triangular, one in compression and the other in tension.

For this case to take place, there must be a certain relation between the steel profile width (t), the section height (h) and the concrete covering (rec).

$$\frac{60}{102}t >= \frac{9}{51}h - \frac{60}{51}rec$$

Note: *See demonstration CE.1 in Annex 1. p. 75*

From the same demonstration it can be seen that the following ratios arise in order to calculate C_{mn} and C_{mx}:

$$C_{mn} = 30(\tfrac{d_2}{51})$$
$$C_{mx} = 30(\tfrac{d_1}{9})$$

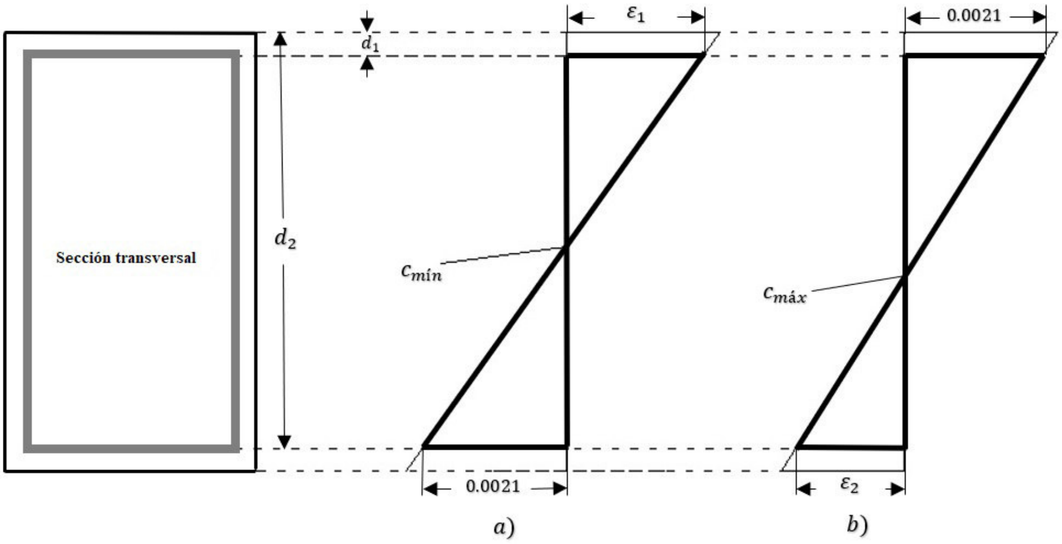


Figure 3.7: *Geometrical strain configurations for the special case, a) For the position of c at the lower limit of the allowed range, b) For the position of c at the upper limit of the allowed range* ***Own made drawing***

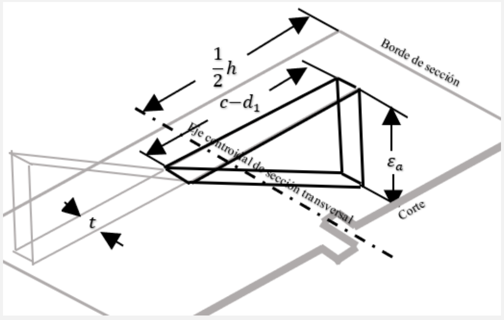
Thus, making reference to **Figure 3.7**, the next unit strain values arise for equations ***Equation 3.2*** and ***Equation 3.3***:

$$\varepsilon_a = (1 - \tfrac{d_1}{c})(0.003)$$
$$\varepsilon_b = (\tfrac{d_2}{c} - 1)(0.003)$$

Y las fuerzas y momentos resistentes se calcularían como:

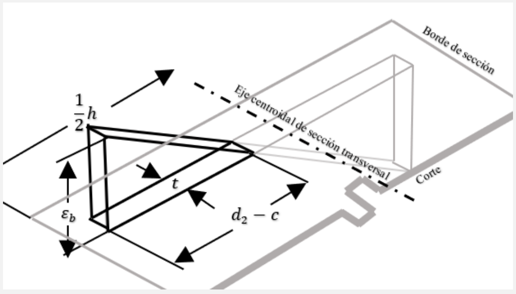
$$F_1 = -\frac{1}{2}\varepsilon_a E_{ac}(c - d_1)2t \qquad (3.27)$$

$$M_1 = -F_1(d_1 + \frac{1}{3}(c - d_1)) \qquad (3.28)$$



$$F_2 = \frac{1}{2}\varepsilon_b E_{ac}(d_2 - c)2t \qquad (3.29)$$

$$M_2 = F_2(d_2 - \frac{1}{3}(d_2 - c)) \qquad (3.30)$$



$$F_R = F_1 + F_2 - F_a + F_b \qquad (3.31)$$

$$M_R = M_1 + M_2 - F_a(d_1 - \frac{1}{3}h) + F_b(d_2 - \frac{1}{2}h) \qquad (3.32)$$

3.1.6 Case 3-4: Four strain geometrical configurations take place, one triangular and one rectangular in compression, and similarly for tension.

$\frac{30}{9}d_1 < c <= \frac{30}{51}d_2$

Let us make reference to **Figure 3.6** to visualize this range.

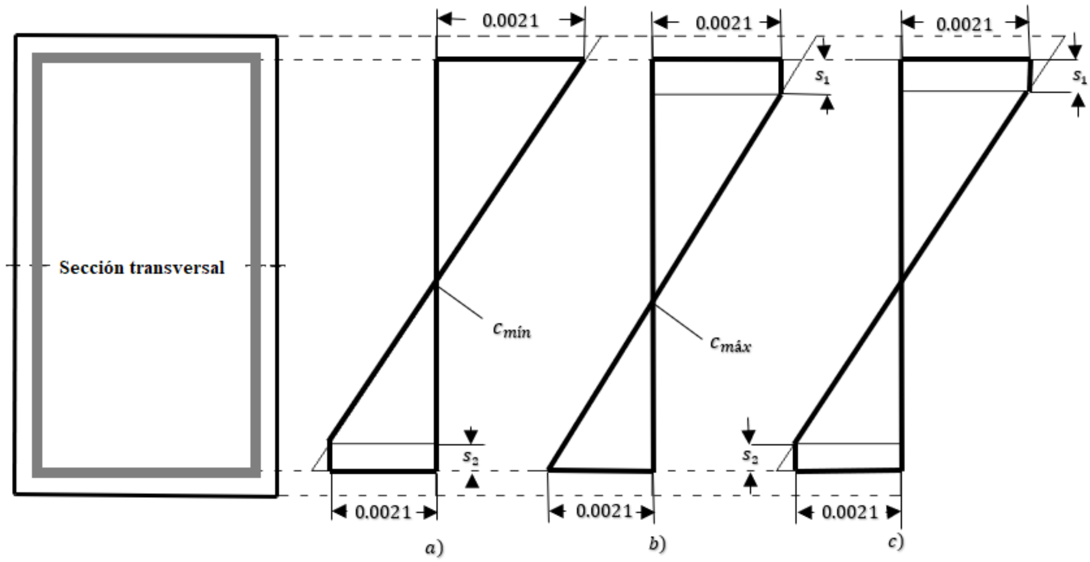


Figure 3.8: *a) Geometrical strain configurations for the position of c at the lower boundary of the allowed range, b) Geometrical strain configurations for c at the upper boundary of the allowed range, c) Geometrical strain configurations for any position of c inside its allowed range. Case 3-4.*

Where:

$$s_1 = \frac{9}{30}c - d_1$$
$$s_2 = h - \frac{51}{30}c - d_1$$

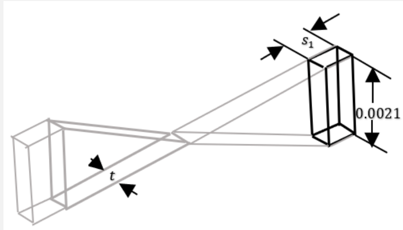
Thus, the following strain values arise for equations ***Equation 3.2*** and ***Equation 3.3***:

$$\varepsilon_a = 0.0021$$
$$\varepsilon_b = 0.0021$$

Hence, to calculate the net resultant normal forces and bending moments, the following equations take place:

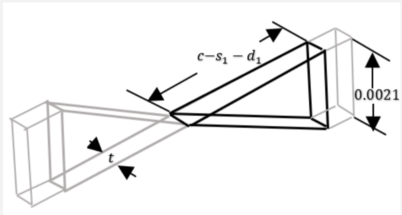
$$F_1 = -2t(0.0021)(s_1)E_{ac} \tag{3.33}$$

$$M_1 = F_1(d_1 + \frac{1}{2}(s_1) - \frac{1}{2}h) \tag{3.34}$$



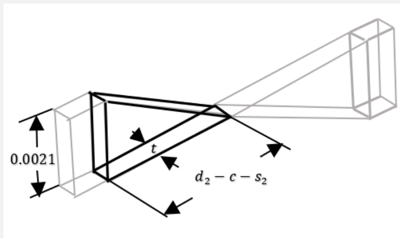
$$F_2 = -\frac{1}{2}2t(0.0021)(c - s_1 - d_1)E_{ac} \tag{3.35}$$

$$M_2 = F_2(d_1 + s_1 + \frac{1}{3}((c - d_1 - s_1)) - \frac{1}{2}h) \tag{3.36}$$



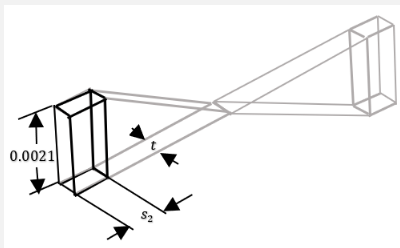
$$F_3 = \frac{1}{2}2t(0.0021)(d_2 - c - s_2)E_{ac} \tag{3.37}$$

$$M_3 = F_3(d_2 - s_2 - \frac{1}{3}(d_2 - c - s_2) - \frac{1}{2}h) \tag{3.38}$$



$$F_4 = 2t(0.0021)(s_2)E_{ac} \tag{3.39}$$

$$M_4 = F_4(d_2 - \frac{1}{2}s_2) - \frac{1}{2}h) \tag{3.40}$$



$$F_R = F_1 + F_2 + F_3 + F_4 - F_a - F_b \tag{3.41}$$

$$\begin{aligned} M_R &= M_1 + M_2 + M_3 + M_4 \\ &-F_a(d_1 - \frac{1}{2}h) - F_b(d_2 - \frac{1}{2}h) \end{aligned} \tag{3.42}$$

3.1.7 Case 4: Three geometrical strain configurations take place, one triangular and one rectangular in compression, as well as one triangular in tension.

$\frac{30}{51}d_2 < c <= d_2$

Let us make reference to **Figure 3.9** for the calculation of the net resistant normal forces and bending moments..

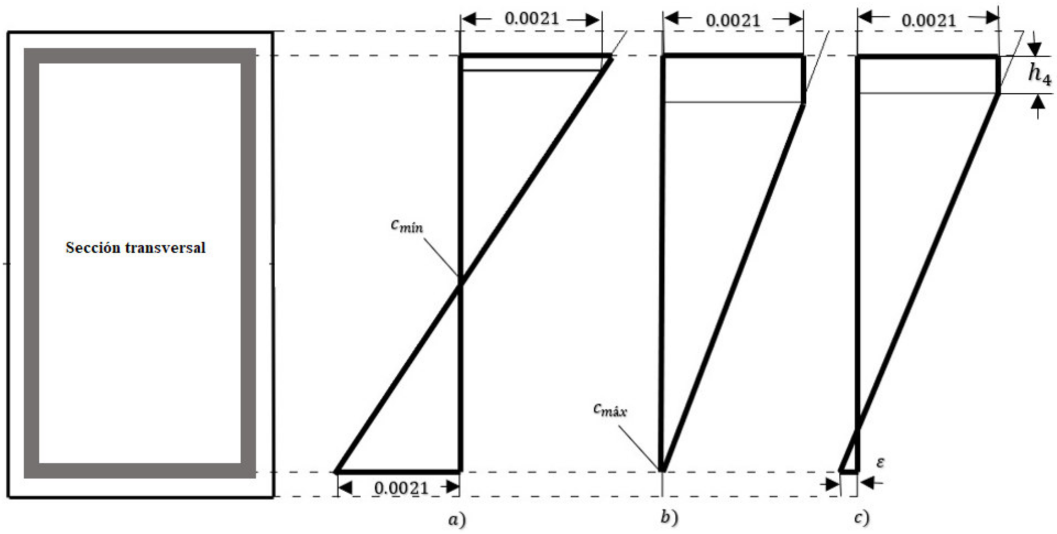


Figure 3.9: *a) Geometrical strain configuration for the position of c at the lower boundary of of its allowed range, b) Geometrical strain configuration for the position of c at the upper limit of the allowed range, c) Geometrical strain configuration for any position of c inside its allowed range. Case 4.*

Where:

$$h_4 = (1 - \frac{21}{30} - \frac{d_1}{c})c$$
$$\varepsilon = 0.003(\frac{d_2}{c} - 1)$$

Nota: *See demonstration 4.1 in Annex 1 p. 76*

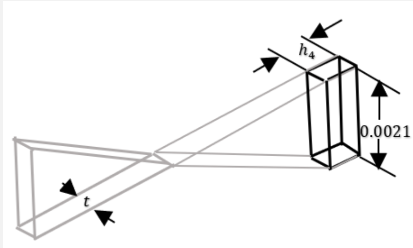
Thus, the following unit strain values arise for equations ***Ecuación 3.2*** and ***Ecuación 3.3***:

$$\varepsilon_a = 0.0021$$
$$\varepsilon_b = \varepsilon$$

And hence, for the calculation of the net resistant normal forces and bending moments, the following equations take place:

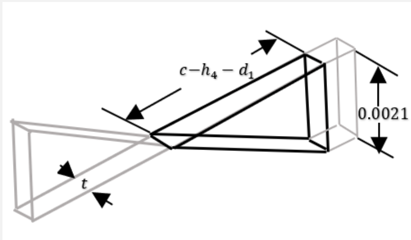
$$F_1 = -2t(0.0021)h_4E_{ac} \tag{3.43}$$

$$M_1 = F_1((d_1 + \frac{1}{2}h_4) - \frac{1}{2}h) \tag{3.44}$$



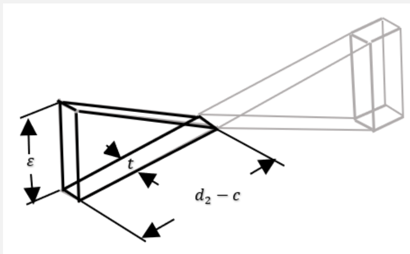
$$F_2 = -\frac{1}{2}2t(0.0021)(c - d_1 - h_4)E_{ac} \tag{3.45}$$

$$M_2 = F_2(\frac{2}{3}(d_1 + h_4 + \frac{1}{2}c) - \frac{1}{2}h) \tag{3.46}$$



$$F_3 = \frac{1}{2}2t\varepsilon(d_2 - c)E_{ac} \tag{3.47}$$

$$M_3 = F_3((\frac{1}{3}c + \frac{2}{3}d_2) - \frac{1}{2}h) \tag{3.48}$$



$$F_R = F_1 + F_2 + F_3 - F_a + F_b \tag{3.49}$$

$$M_R = M_1 + M_2 + M_3 - F_a(d_1 - \frac{1}{2}h) + F_b(d_2 - \frac{1}{2}h) \tag{3.50}$$

3.1.8 Case 5. Three geometrical strain configurations are presented; two rectangular and one triangular, all in compression.

$d_2 < c <= \frac{30}{9} d_2$

Let us make reference to **Figure 3.10** to visualize this range, and to the demonstration indicated in the figure caption.

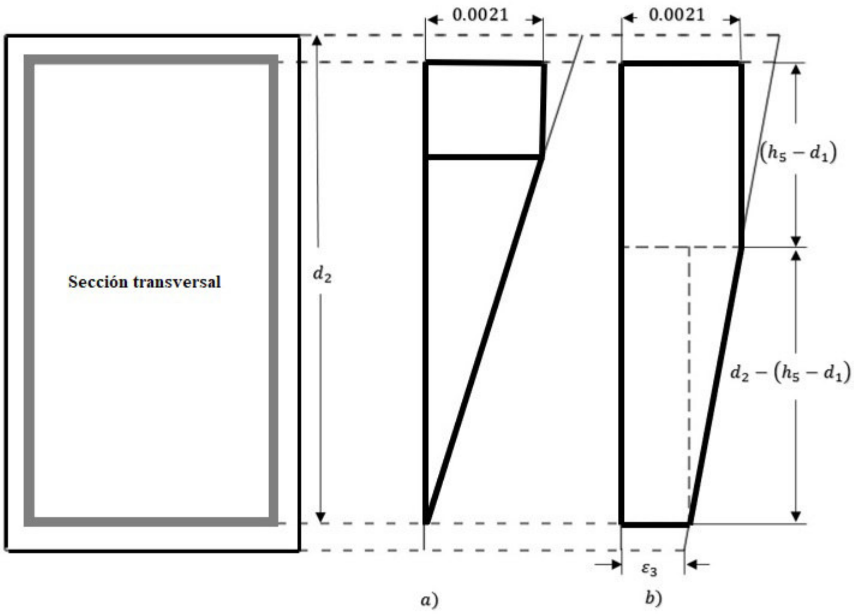


Figure 3.10: *a) Geometrical strain configurations at the lower boundary of the allowed range, b) Geometrical strain configuration for the position of c at the upper boundary of the allowed range. Case 5.* ***Own made drawing.*** *See demonstration 5.1 in Annex 1 p. 77*

.

Where:

$$h_5 = \frac{9}{30} c$$
$$\varepsilon_3 = 0.0021 - \frac{0.003}{c}(d_2 - h_5)$$

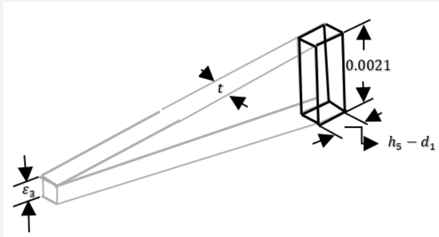
Thus, the following unit strain values take place for equations ***Equation 3.2*** and ***Equation 3.3***:

$$\varepsilon_a = 0.0021$$
$$\varepsilon_b = \varepsilon_3$$

Hence, for calculation of the net resistant forces and bending moments, the following equations arise:

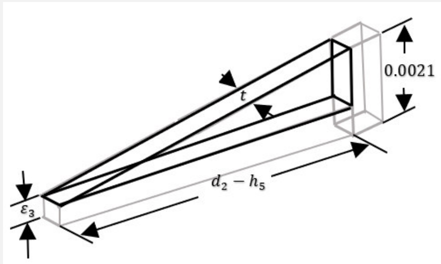
$$F_1 = -2t(0.0021)(h_5 - d_1)E_{ac} \quad (3.51)$$

$$M_1 = F_1(\frac{1}{2}(h_5 + d_1) - \frac{1}{2}h) \quad (3.52)$$



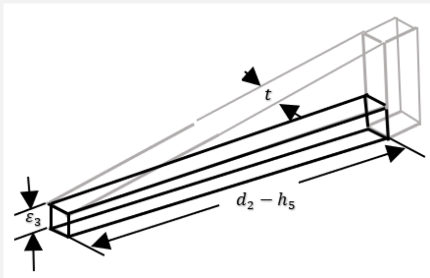
$$F_2 = -\frac{1}{2}2t(0.0021 - \varepsilon_b)(d_2 - h_5)E_{ac} \quad (3.53)$$

$$M_2 = F_2(\frac{2}{3}(h_5 + \frac{1}{2}d_2) - \frac{1}{2}h) \quad (3.54)$$



$$F_3 = -\varepsilon_b(d_2 - h_5)2tE_{ac} \quad (3.55)$$

$$M_3 = F_3(\frac{1}{2}(d_2 + h_5) - \frac{1}{2}h) \quad (3.56)$$



$$F_R = F_1 + F_2 + F_3 - F_a - F_b \quad (3.57)$$

$$M_R = M_1 + M_2 + M_3 - F_a(d_1 - \frac{1}{2}h) - F_b(d_2 - \frac{1}{2}h) \quad (3.58)$$

3.1.9 Case 6. One rectangular geometrical strain configuration in compression takes places.

$c > \frac{30}{9} d_2$

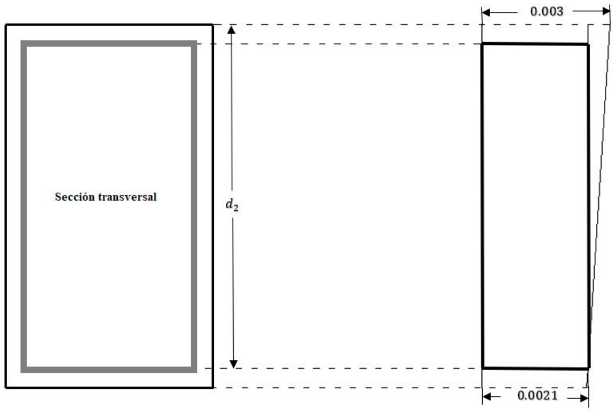
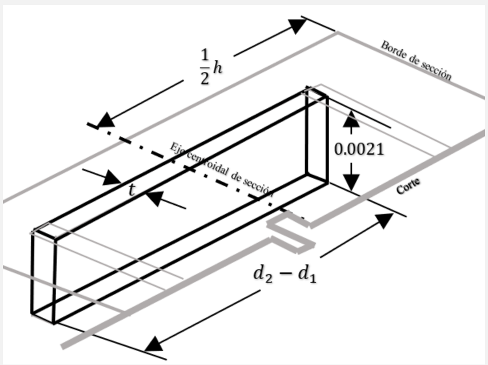


Figure 3.11: *Geometrical strain configurations in pure compression (the netrual axis c position is located right at the inferior boundary of its allowed range). Case 6.* ***(Own made drawing)****, see demonstration 6.1 in Annex 1. 77*

For this case, the cross section will only withstand normal compression forces, **Figure 3.11**, as the generated geometrical configuration has its centroid right at the middle of the cross section itself.

Hence:



$$F_1 = -0.0021(d_2 - d_1)E_{ac}2t \tag{3.59}$$

$$F_R = F_1 - F_a - F_b \tag{3.60}$$

$$M_R = 0 \tag{3.61}$$

3.1.10 Analysis cases for concrete

What is pretended here is to seek the net area of the concrete cross section subjected to compression stresses for any given value of the neutral axis position c **Figure 3.13**. Given the rectangular geometry of the cross section, the analysis may be simplified to a just one single case, as the compression area varies linearly along the cross section height, either with respect of the X axis or the Y axis. The net volume of the generated rectangular geometrical strain block may be estimated then, from that given value of the compression area in force units,and later the resultant net bending moment with respect of the centroid concrete cross section, as following:

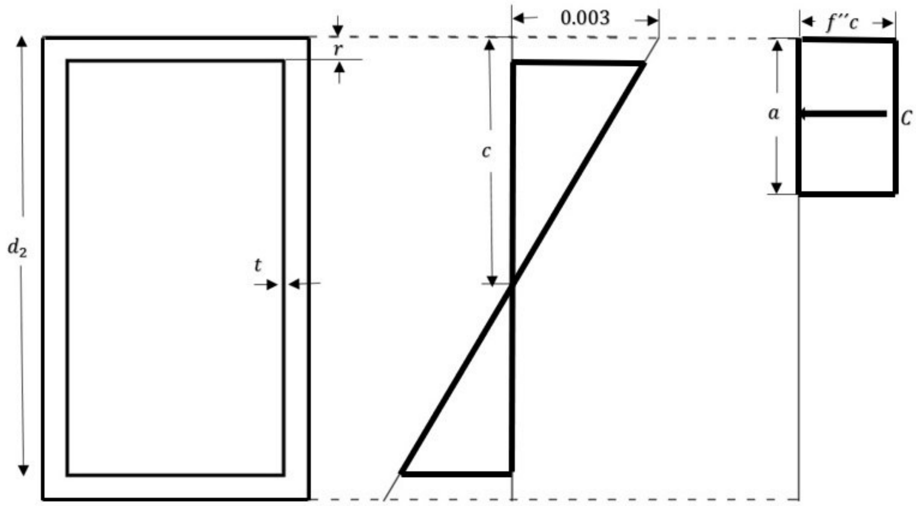


Figure 3.12: *Equivalent compression stresses block for concrete.* ***Own made drawing.***

$$C = -abf''c \tag{3.62}$$

$$M_c = -C(\frac{h}{2} - \frac{a}{2}) \tag{3.63}$$

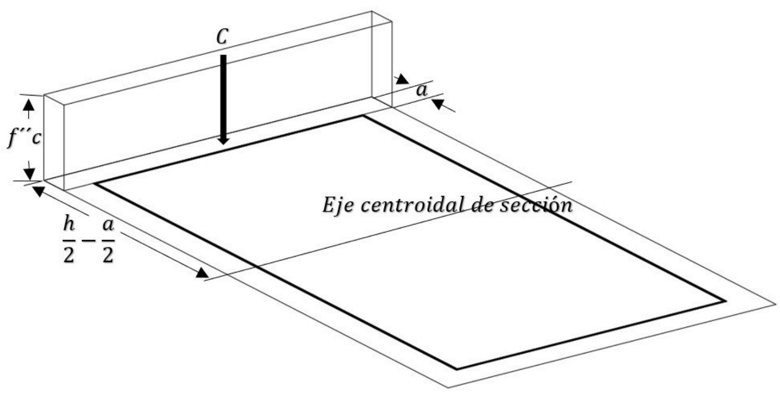


Figure 3.13: *Equivalent 3D compression stresses block for concrete.* ***Own made drawing.***

3.2 Circular columns

Analogous to the idealized reinforcing steel profile for rectangular columns, a circular reinforcing steel profile has been made up as well as shown in the following figure **Figure 3.14**.

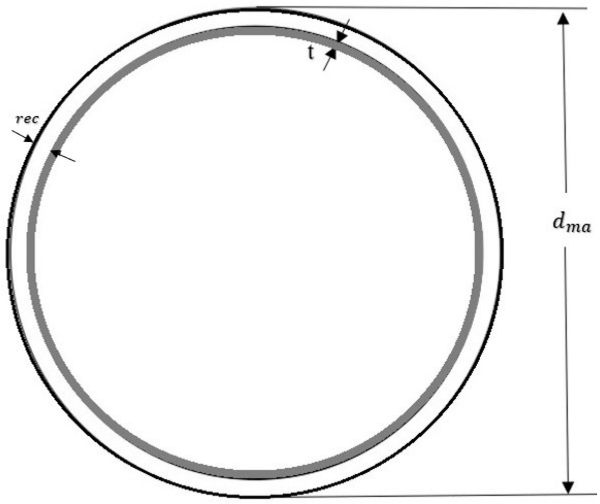


Figure 3.14: *Idealized reinforced circular concrete column cross section.* ***Own made drawing.***

Where:

t = Steel profile width
rec = concrete covering
d_{ma} = element cross section diameter

Note: *The concrete covering value is measured from the concrete cross section edge to the middle of the steel profile width.*

Unlike to rectangular geometries, for circular cross sections it resulted quite complex the strain distribution analysis through the original approach (determining the resulting geometry configurations on any given strain state), hence an alternative approach was made up instead.

3.2.1 Calculus of reinforcing steel resistance

The intended approach will be to treat the reinforcing steel profile as a linear circular line divided in small segments of with t, **Figure 3.15**, although each segment will actually be idealized as a point to which certain strain condition will be assigned depending on its very location over the cross section at a given position of c along it.

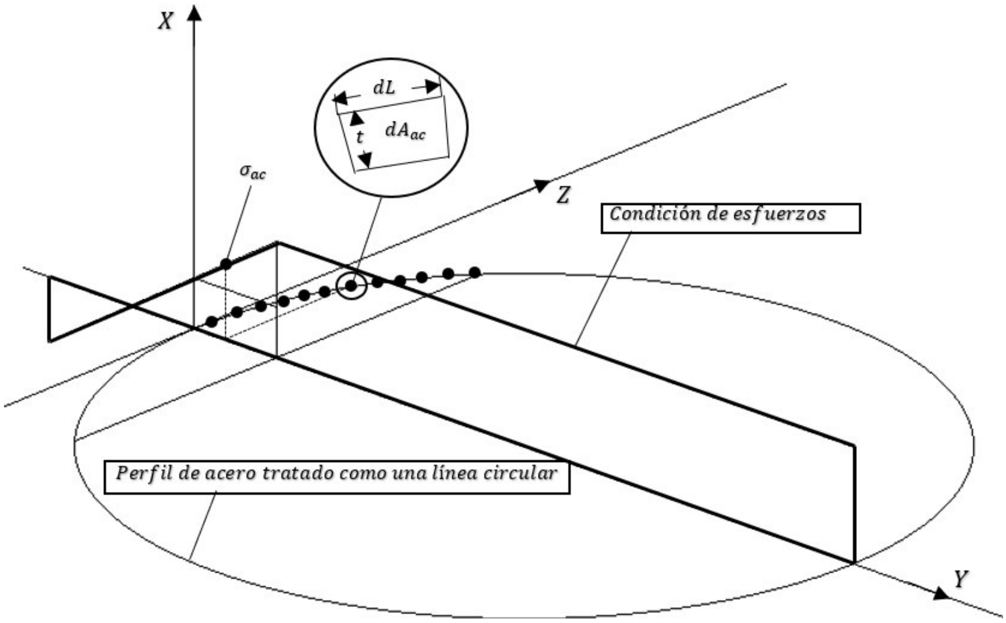


Figure 3.15: *Reference plane for the reinforcing steel strain condition.* ***Own made drawing.***

Where:

$$dA_{ac} = dL(t)$$
$$L = \Pi(d_{ma} - 2rec)$$
$$dL = \frac{L}{n}$$
$$n = Number\ of\ elements$$

Methodology

To determine the axial force and bending moment resistance, the following methodology might be taken as a guide:

- To determine the distance of each steel profile segment with respect to the outermost fibre of the concrete cross section along the the variable direction of c d.
- With d already known, it proceeds to calculate ε for the given segment in function of c, taking care solely of the following restriction $[-0.0021 < \varepsilon < 0.0021]$.
- And finally, the net resistant normal force and bending moment is estimated as following:

$$F_R = \sum_{i=1}^{nElementos} E_{ac}\varepsilon(dA_{ac}) \tag{3.64}$$

$$M_R = \sum_{i=1}^{nElementos} -E_{ac}\varepsilon(dA_{ac})(\frac{1}{2}d_{ma} - d) \tag{3.65}$$

3.2.2 Analysis cases for concrete

Unlike rectangular cross section columns, for circular cross sections the differential of concrete area undergoing compression stress along the variation of c is not uniform.

The concrete area subjected to compression could also be transformed into an equivalent stress block, and in order for this to be possible, it is necessary to calculate first the the real compression area centroid at every position of c, and from there on to just determine the resistant net normal force and bending moment for the concrete cross section. ***Figure 3.16.***

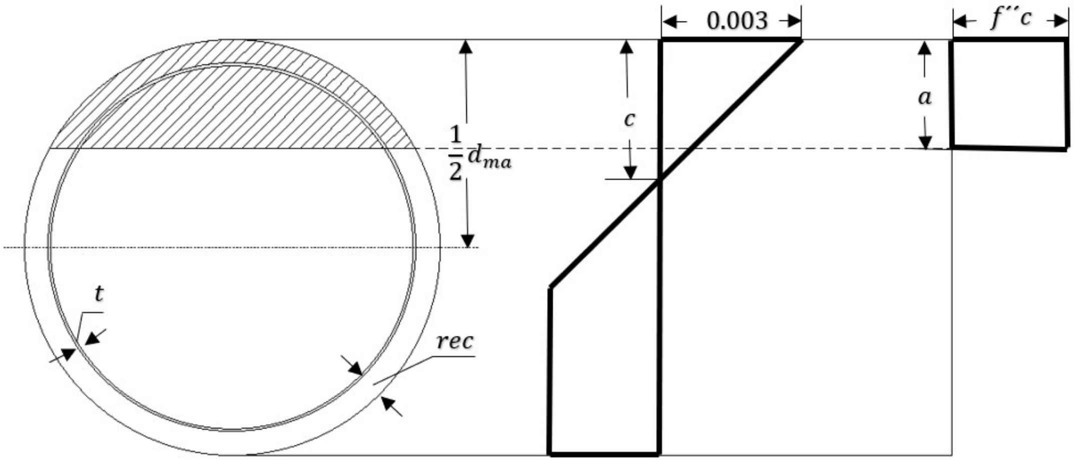


Figure 3.16: *Strain distribution diagram for the concrete element cross section.* ***Own made drawing.***

Where:

$$a = \beta(c)$$
$$f''_c = F_R(0.85)f'c$$

Let us take reference to the next figure **Figure 3.17** to visualize better the compression area for the concrete section, to be able to define the integrals for the calculation of the effective concrete area under compression at any given position of the neutral axis c.

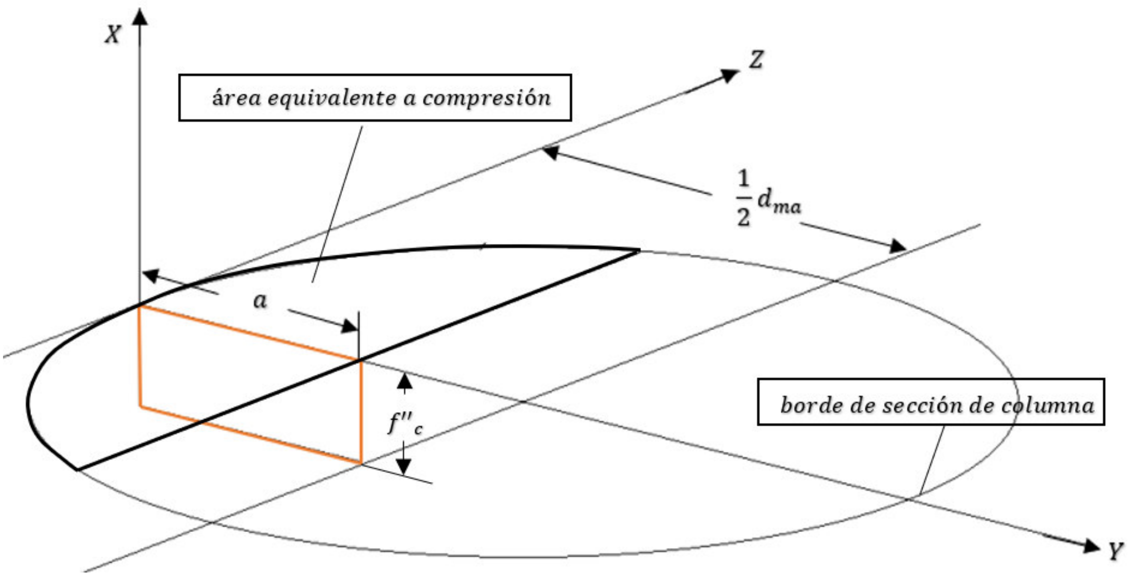


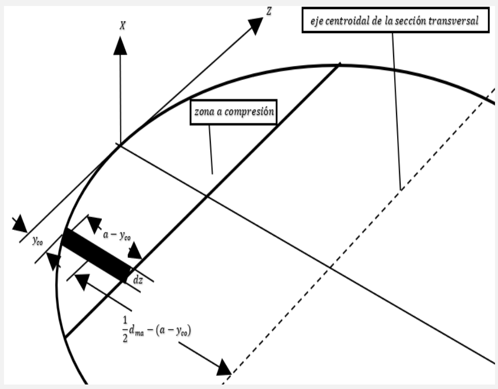
Figure 3.17: *Cartesian system of reference for the strain distribution analysis.* ***Own made drawing.***

The analysis will be divided into the following two cases of analysis for simplification:

When $a < \frac{1}{2}d_{ma}$

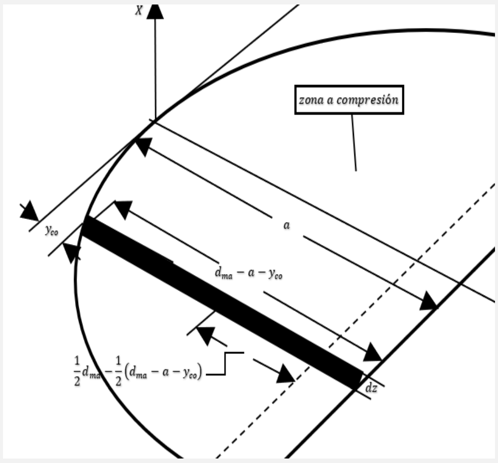
$$F_{co} = f''c \int_{-z(y_{co}=a)}^{z(y_{co}=a)} (a - y_{co})\mathrm{d}z \quad (3.66)$$

$$F_{co} \cdot Y_{co} = f''c \int_{-z(y_{co}=a)}^{z(y_{co}=a)} (a - y_{co})\ldots \left(\frac{1}{2}d_{ma} - \frac{1}{2}(a + y_{co})\right)\mathrm{d}z \quad (3.67)$$



When $a >= \frac{1}{2}d_{ma}$

$$F_{co} = f''c(\Pi(\frac{1}{2}d_{ma})^2 - ...$$
$$... - \int_{-z(y_{co}=d_{ma}-a)}^{z(y_{co}=d_{ma}-a)} ((d_{ma} - a) - y_{co})\mathrm{d}z) \tag{3.68}$$



$$F_{co} \cdot Y_{co} = f''c(-\int_{-z(y_{co}=d_{ma}-a)}^{z(y_{co}=d_{ma}-a)} ((d_{ma} - ...$$
$$... - a) - y_{co})(-(\frac{1}{2}d_{ma} - ...$$
$$... - \frac{1}{2}(d_{ma} - a - y_{co}))\mathrm{d}z) \tag{3.69}$$

4. Program

4.1 Geometry and mechanic properties of materials

Recalling the main objective of the project which is to determine a reinforcing bar arrangement such that it may comply the required acceptable structural efficiency before certain loads the column itself might be subjected to, keeping its initially proposed cross section geometry as constant through the calculations. Thus, the entry data will be of course, the number of columns to design, and for each of this elements its simple compression resistance f'_c will be indicated, as well as its concrete covering *rec* and in a general way the steel creep stress factor for the reinforcing bars used f_y.

Therefore, the program will also require the acting loads over each column for its design. Such acting loads are previously determined through a structural analysis using commercial software for that purpose or with one own costumed by the designer. Another important entry data is the initial value for the steel profile width t, which will be accommodated at the concrete cross section, respecting the required concrete covering, in order to start the calculations and iterations of the analysis, updating the interaction diagrams for each columns until the most critical mechanic load is covered.

4.2 Interaction diagrams calculation

A certain number of points to determine the interaction diagrams is established initially, estimating the values forP_{ot}, P_{oc} as the limits of the normal resistant forces range for the construction of the diagram, making possible to extract a respective force value to be assigned to each point. This way a numerical method can be applied, taking the value of each point as a root seeking its respective bending moment resistant value.

The numerical method hereby employed for the roots approximation is the one so called "*False position* method". [5]

4.2.1 Calculus for the structural-mechanical efficiency

For each acting load over a column, a respective structural efficiency between this such load condition and the resistance the interaction diagram indicates.

The very difference between circular columns and rectangular columns for the calculations of the structural efficiencies is that for rectangular geometries there must be an extraction of two moment pair values ($M_{rx}, M_{ry}, P_{rx}, P_{ry}$, each moment pair taken from its respective axis interaction diagrams, x and y, while on the other hand for circular geometries, there will only be one solely moment pair M_r, P_r calculated for each load condition efficiency, taking on account that their cross section is symmetrical with respect of any axis.

Thus, analytical geometry will be employed using the Cartesian system of reference of the interaction diagrams themselves, **Figure 4.1**. Therefore, to start with, the intersection point between two imaginary straight lines A and B will be determined, such that A will go from the origin passing by a load condition (M_u, P_u) and prolonging until the interaction diagram border, and B will be such that it will join a previous point (M_i, P_i) and a subsequent one (M_{i+1}, P_{i+1}) with respect of the point to which line A intersects with the interaction diagram (M_r, P_r).

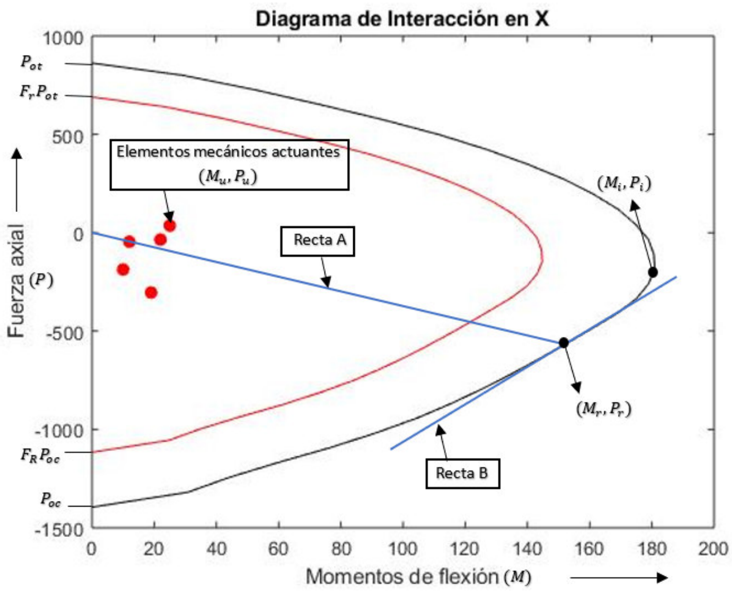


Figure 4.1: *Interaction diagram in the Cartesian plane of reference for the program.*

Hence, the resistant bending moment for a respective load condition will be calculated as following:

$$M_r = \frac{P_{i+1} + \left(\frac{P_i - P_{i+1}}{M_{i+1} - M_i}\right)}{\frac{P_u}{M_u} - \frac{P_{i+1} - P_i}{M_{i+1} - M_i}} \tag{4.1}$$

And the resistant normal force as:

$$P_r = \frac{P_u}{M_u} M_r \tag{4.2}$$

See demonstration p. 78

Now, the formulas from NTC-2017 p. 31, to calculate efficiencies can be applied.

4.2.2 Calculation of the required reinforcing steel area

Once determined the structural efficiency for each load condition, the most critical is then located, and from this such critical condition is with which the interaction diagrams are updated on each iteration changing the value of the steel profile width t with a certain differential increase dt. When the required structural efficiency is complied then a approximated required reinforcing steel area has been finally found. **Figure [4.2]**

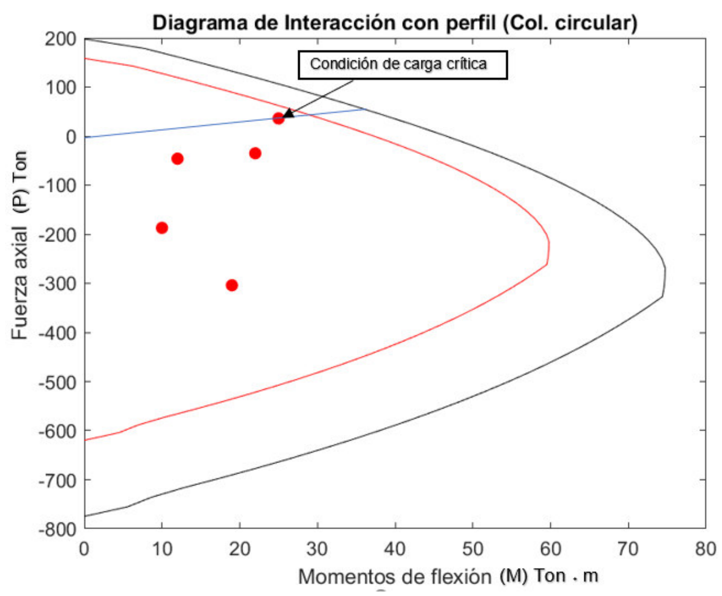


Figure 4.2: *Critical design load condition.*

4.3 Determination of steel reinforcing bars

Once determined the required steel area to cover the most critical load condition, it must be transformed into reinforcing steel bars, making a proper and uniform distribution over the cross section (according to the design hypothesis and applicable standard norms) ***Section 2.3.3***), thus, only pair numbers of reinforcing bars will be allowed.

The program will then generate a matrix arrangement of proposals for number of bars of a respective commercial diameter, varying from 4 (for rectangular columns) or 6 (for circular columns), covering always the previous estimated required steel area.

Thus, the user will be able to choose not only the most convenient bar diameter but also the distribution of such bars over the element cross section (only for rectangular columns), respecting at all times the allowed separation between each bar $S_{min} = \frac{3}{2}TMA$.

Worth to mention is that the very distribution of reinforcing bars over the cross section might have great influence over the structural efficiency of the element, and for each option of bar/diameter will correspond a great number of various configurations of them over the cross section, thus, it was though recommendable that the program did as well an efficiency analysis of each available option, but considering now the reinforcing steel as bars accommodated over the cross section, in order to verify indeed, and more precisely which ones complied the required efficiency range.

4.4 Employed programming platforms and languages

With the purpose of giving the project more portability, and to the program in particular, it was written in C Language, being such language compatible with many other software and applications, such as Qt Creator, to design graphic interfaces to make the program more user-friendly. With this language there's also the advantage of making it of easy adaptation for other languages such as Python, using C Python API, that in turn has a lot of reachable applications such as PyRevit. And not to mention the speed of execution of the code in C, and moreover that the code itself may be manipulated to apply Parallel Computing with OpenMP for instance, for massive tasks.

MatLab was also used, taking advantage of their great graphic functions to graph the interaction diagrams and drawing the reinforced concrete columns cross sections, importing the results from the previous made analysis in C language.

4.5 Input data

As input data, .txt files are generated with information such as the dimensions of each column, the concrete covering, as well as the number of columns to be designed **Table 4.5.1** and **Table 4.5.3** and of course the load conditions to which each columns is subjected taken from the previous structural analysis **Table 4.5.2** and **Table 4.5.4**

4.5.1 Rectangular columns

$b(cm)$	$h(cm)$	$rec(cm)$	$f'c$
<>	<>	<>	<>

Table 4.5.1: *Input data form table for the program of rectangular columns*

$P_u(Ton)$	$Mu_x(Ton \cdot m)$	$Mu_y(Ton \cdot m)$
<>	<>	<>

Table 4.5.2: *Input data table form of load conditions for rectangular columns*

4.5.2 Circular columns

$d_{ma}(cm)$	$rec(cm)$	$f'c$
<>	<>	<>

Table 4.5.3: *Input data table form of geometry for circular columns*

$P_u(Ton)$	$Mu(Ton \cdot m)$
<>	<>

Table 4.5.4: *Input data table form of load conditions for circular columns*

4.6 Information output

As output results it will obtained: the resulting efficiencies for each available reinforcing bar option **Tables [4.6.1],[4.6.2]**, the interaction diagrams points both for the final reinforcing steel profile **Figures [4.4]** and for the chosen arrangement of reinforcing bars **Figures [4.6],[4.7]**, as well as the coordinates of the reinforcing bars over the cross section, and the cross section border points **Figures [4.3]**, separated in folders for each designed column, to be read later (with MatLab in this case) to construct the graphs and reinforced section drawings, importing this such information as .txt files format.

4.6.1 Rectangular columns output

For this geometry, a diagram for each axis direction X and Y, is obtained.

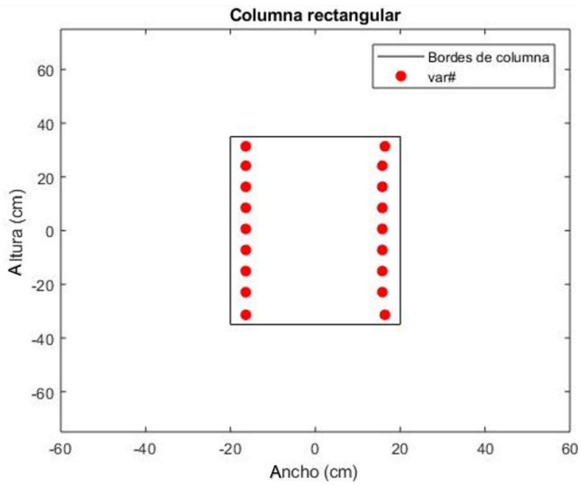


Figure 4.3: *Reinforced concrete cross section for a designed rectangular column.*

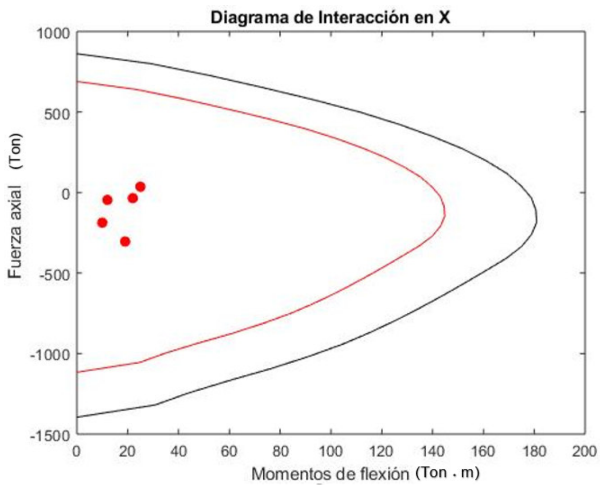


Figure 4.4: *Graphed interaction diagram for the chosen reinforcing bar arrangement respect to the X axis.*

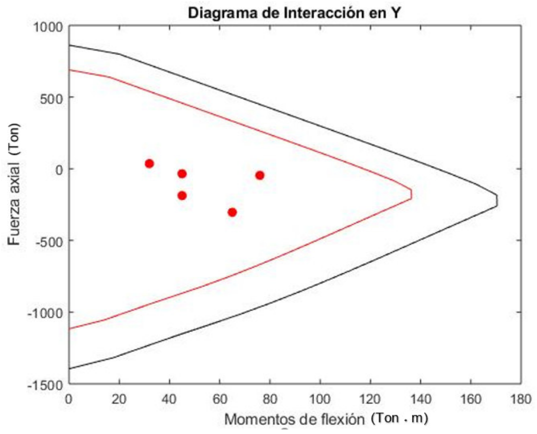


Figure 4.5: *Graphed interaction diagram for the chosen reinforcing bar arrangement respect to the Y axis.*

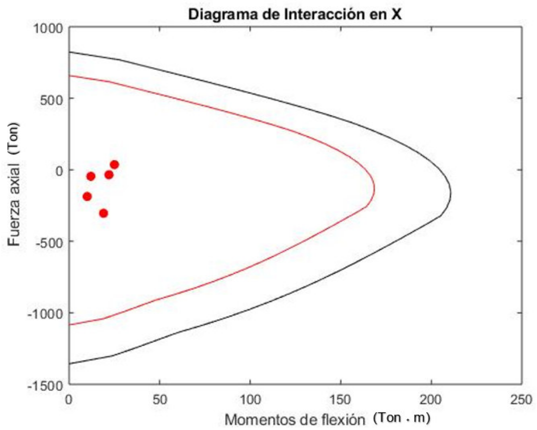


Figure 4.6: *Graphed interaction diagram for final reinforcing steel profile respect to the X axis.*

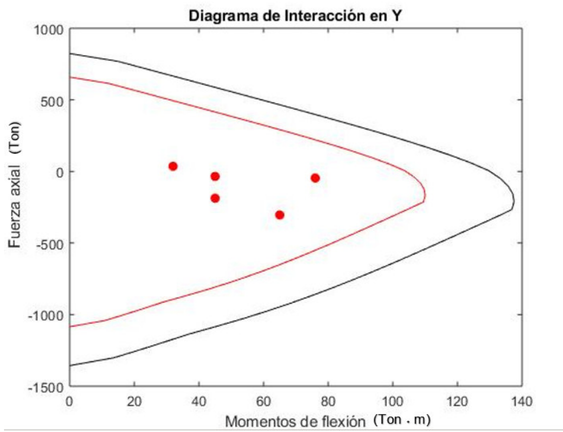


Figure 4.7: *DGraphed interaction diagram for final reinforcing steel profile respect to the Y axis.*

$Pr(Ton)$	$Mr_x(Ton \cdot m)$	$Mr_y(Ton \cdot m)$	$Ef(\%)$
<>	<>	<>	<>

Table 4.6.1: *Resulted table form with the critical efficiencies for each available reinforcing bar option for rectangular columns.*

4.6.2 Circular columns

As mentioned before, just one interaction diagram is obtained for each column, given the symmetry with respect of any orientation of its section centroid axis.

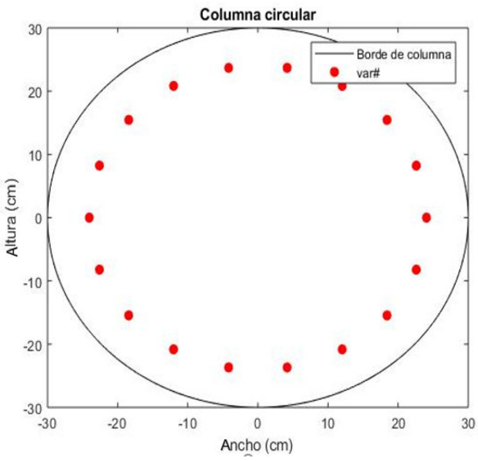


Figure 4.8: *Drawing of the designed reinforced concrete column cross section.*

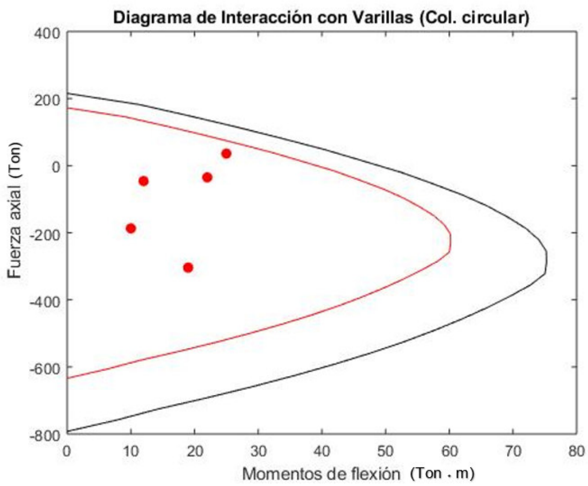


Figure 4.9: *Interaction diagram of the reinforced column with the chosen reinforcing bar.*

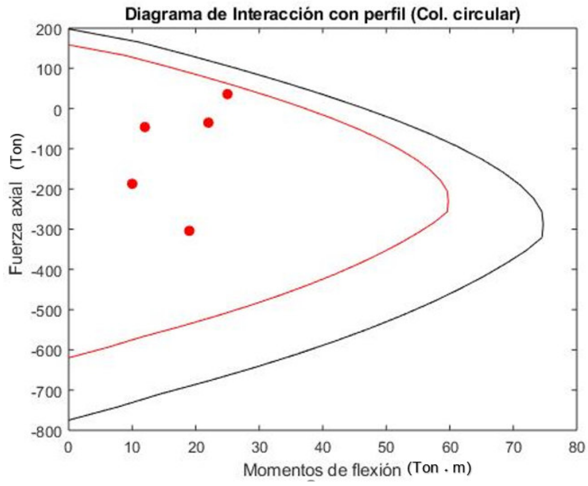


Figure 4.10: *Interaction diagram for the reinforced concrete column with the final reinforcing steel profile.*

$P_u(Ton)$	$Mu(Ton \cdot m)$	$Pr(Ton)$	$Mr(Ton \cdot m)$	$Ef(\%)$
<>	<>	<>	<>	<>

Table 4.6.2: *Resulting table form for the efficiencies of each available reinforcing bar arrangement for circular columns.*

4.7 General pseudo code of the program

Algorithm 1: Interaction diagram calculation

BEGINNING

$As = (2*(b-2rec)+2*(h-2rec))t$

$poc = f''c(Ac-As)+fy(As)$

$pot = As(fy)$

$df = (pot-poc)/(Npoints)$

Do i=1,Npoints

$P(i) = -Fr(poc+i*df)$

End Do

Do i=1,Npoints

$M(i) = casosAnalisis(P(i),a,c,h,b,t,rec,E)$

End Do

Algorithm 2: Eficciency calculation for the load conditions of the idealized cross section

Do i=1,Nconditions

$\frac{1}{e(u)} = \frac{Mu(i)}{Pu(i)}$

$[Pr(i),Mr(i)] = resistance(\frac{1}{e(u)},condition)$

End Do

$eficiencia = \frac{Mu}{Mr}$

If (maxEfficiency<limMinor) then

$t = t-dt$

Return to **Algorithm 1**

elseif (maxEfficiency>limMayor))

$t = t+dt$

Return to **Algorithm 1**

else

END

End If

Algorithm 3: Determination of the reinforcing bar options

BEGINNING
Do i=1,NbarOptions
nBars=1
Asv=areaBar(i)*nBar
Do while (Asv<As)
nBar=nBar+1
Asv=areaBar(i)*nBar
End Do
End Do

Algorithm 4: Calculation of the interaction diagram for the chosen reinforcing bar option

Selection of reinforcing bar option
$poc = f''c(Ac - Asv) + fy(Asv)$
$pot = Asv(fy)$
$df = (pot - poc)/(Npoints)$
Do i=1,Npoints
$P(i) = -Fr(poc + i * df)$
End Do

Do i=1,Npoints
$M(i) = elemMechanic(P(i), a, c, h, b, numeroBars, areaBar, rec, E)$
End Do

Algorithm 5: Calculation of efficiency for the chosen reinforcing bar option

Do i=1,Nconditions
$\frac{1}{e(u)} = \frac{Mu(i)}{Pu(i)}$
$[Pr(i), Mr(i)] = resistance(\frac{1}{e(u)}, condition)$
End Do
$efficiency = \frac{Mu}{Mr}$
If (maxEfficiency<limMinor or maxEfficiency>limMayor) then
Return to **Algorithm 4**
else
END
End If

Part 3

Annex

4.8 Annex 1. Involved variables for the analysis cases of rectangular columns

Case 1

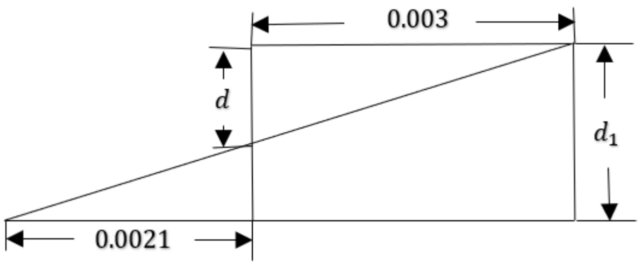


Figure 4.11: *Demonstration 1.1 for Case 1 of rectangular columns.* ***Own made drawing.***

By relationship of similar triangles:

$$\frac{0.003}{d} = \frac{0.0051}{d_1}$$

$$d = \frac{30}{51} d_1$$

Case 2

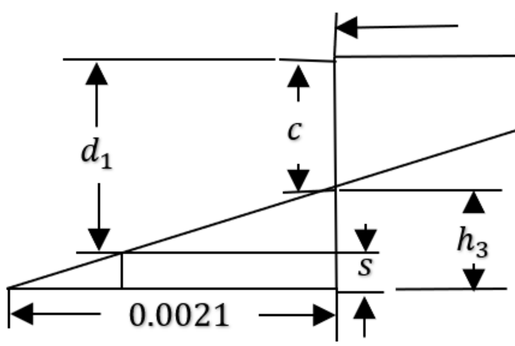


Figure 4.12: *Demonstration 2.1 for Case 2 of rectangular columns.*

$$s = h_3 + c - d_1$$

By relationship of similar triangles:

$$\frac{\varepsilon}{h_3 - s} = \frac{0.003}{c}$$

Then:

$$\varepsilon = \frac{0.003}{c}(h_3 - (h_3 + c - d_1))$$
$$\varepsilon = 0.003(\frac{d_1}{c} - 1)$$

Case 3

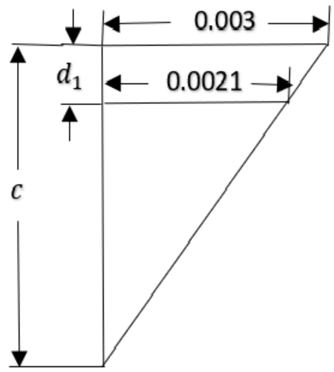


Figure 4.13: *Demonstration 3.1 for Case 3 of rectangular columns*

And by relationship of similar triangles:

$$\frac{c}{0.003} = \frac{d_1}{0.003-0.0021}$$

Then:

$$c = \frac{30}{9}d_1$$

Special case

Let m_{max} be the linear stress distribution ratio when c is at the lower boundary of the special case range, and m_{mn} the linear stress distribution ratio when c is at the upper limit the special case range, then:

$$m_{max} = \frac{d_1}{0.003-0.0021}$$
$$m_{min} = \frac{d_2}{0.003+0.0021}$$

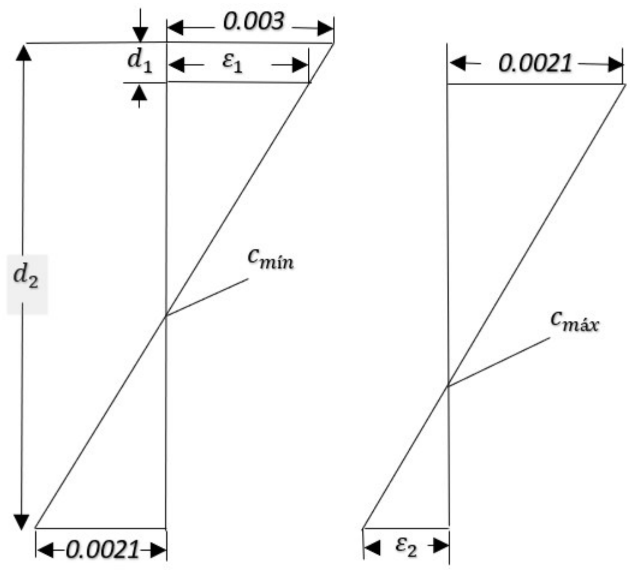


Figure 4.14: *Demonstration CE.1 for the special case of rectangular columns*

Thus, the following ratio should be complied:

$$\frac{d_2}{0.0051} <= \frac{d_1}{0.0009}$$
$$\frac{9}{51}d_2 <= d_1$$

Hence:

$$\frac{9}{51}(h - rec - \frac{1}{2}t <= rec + \frac{1}{2}t)$$

And simplifying:

$$\frac{60}{102}t >= \frac{9}{51}h - \frac{60}{51}rec$$

By trigonometric relationships

$$\frac{c-d_1}{\varepsilon_1} = \frac{c}{0.003}$$
$$\varepsilon_1 = 0.003(1 - \frac{d_1}{c})$$

Besides:

$$\frac{0.003}{c} = \frac{\varepsilon_2}{d_2-c}$$
$$\varepsilon_2 = 0.003(\frac{d_2}{c} - 1)$$

Case 4

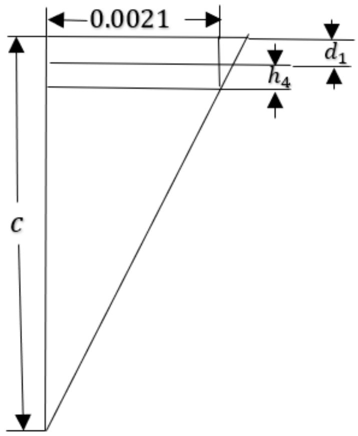


Figure 4.15: *Demonstration 4.1 of Case 4.*

By trigonometric relationships

$$\frac{h_4+d_1}{0.003-0.0021} = \frac{c}{0.003}$$

Hence:

$$h_4 = (1 - \frac{21}{30} - \frac{d_1}{c})c$$

Case 5

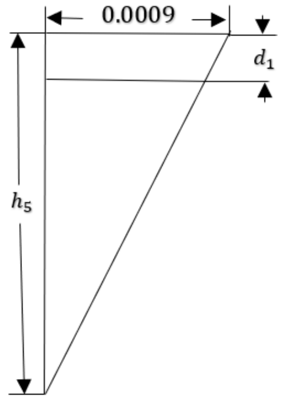


Figure 4.16: *Demonstration 5.1 for Case 5*

By trigonometric relationships:

$$\frac{0.003}{c} = \frac{0.0009}{h_5}$$

Case 6

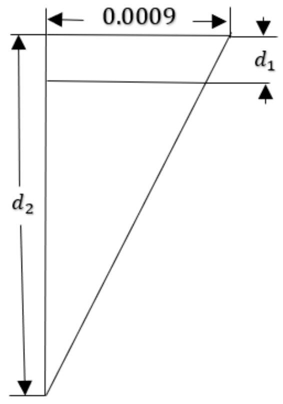


Figure 4.17: *Demonstration 6.1 for Case 6*

By trigonometric relationships:

$$\frac{0.0009}{d_2} \frac{0.003}{c}$$

Then:

$$c = \frac{30}{9} d_2$$

4.9 Annex 2. Demonstration of the program development

4.9.1 Analytical geometry for the calculation of mechanical efficiencies

Equation of straight line A:

$$y = \frac{y_u}{x_u}x \tag{4.3}$$

Equation of straight line B:

$$y = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}x + (y_{i+1} + (\frac{y_i - y_{i+1}}{x_{i+1} - x_i})x_{i+1}) \tag{4.4}$$

Equating both previous equations by compatibility:

$$\frac{y_u}{x_u}x = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}x + (y_{i+1} + (\frac{y_i - y_{i+1}}{x_{i+1} - x_i})x_{i+1}) \tag{4.5}$$

Isolating the variable x from the previous equation:

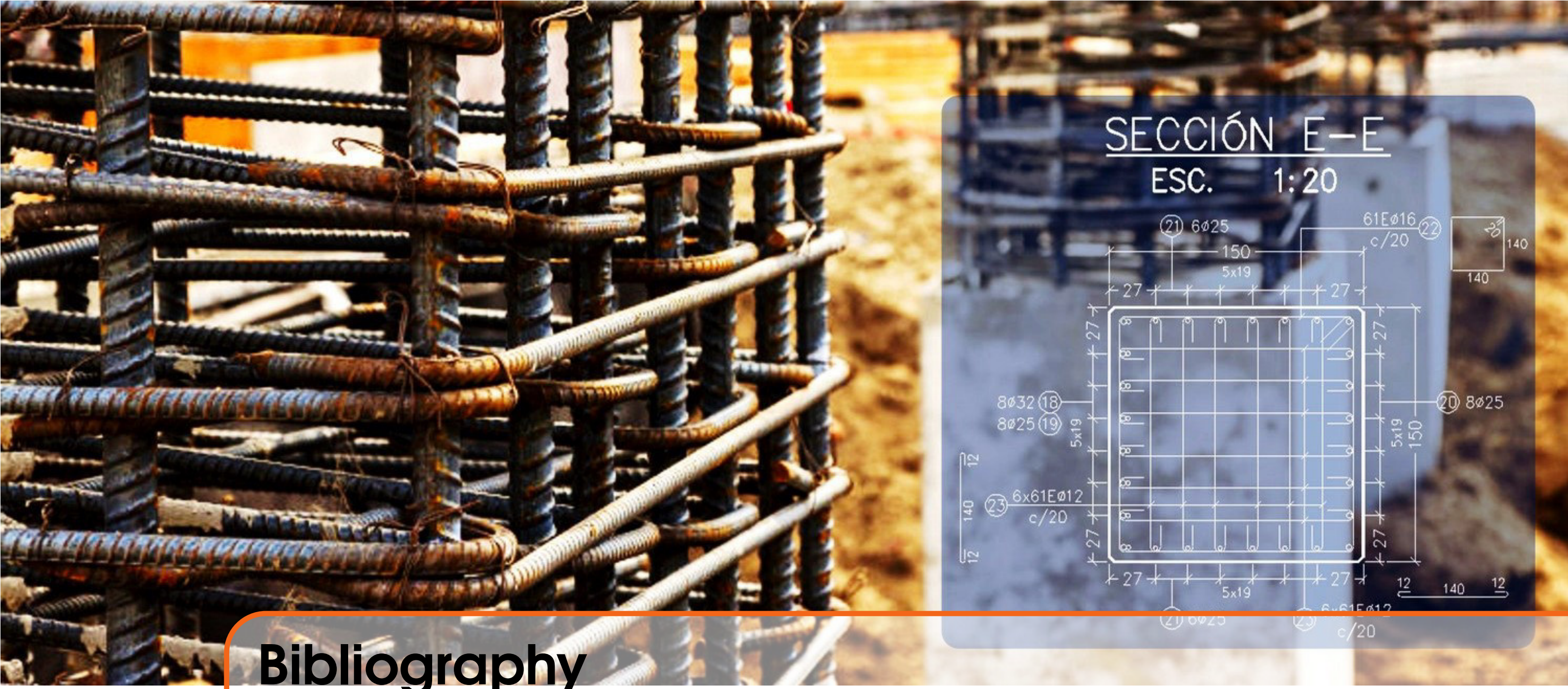
$$x = x_r = \frac{y_{i+1} + (\frac{y_i - y_{i+1}}{x_{i+1} - x_i})}{\frac{y_u}{x_u} - \frac{y_{i+1} - y_i}{x_{i+1} - x_i}} \tag{4.6}$$

Substituting ***Equation [4.6]*** in ***Equation [4.5]***.

$$y = y_r = \frac{y_u}{x_u}x_r \tag{4.7}$$

Where:

$$y_u = P_u$$
$$x_u = M_u$$
$$y_r = P_r$$
$$x_r = M_r$$
$$y_i = P_i$$
$$x_i = M_i$$
$$y_{i+1} = P_{i+1}$$
$$x_{i+1} = M_{i+1}$$



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