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MASTER THESIS

Renormalization of a model for spin-1 matter fields

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Declaration of Authorship

I, Ailier RIVERO ACOSTA, declare that this thesis titled, "Renormalization of a model for spin-1 matter fields" and the work presented in it are my own. I confirm that:

- This work was done wholly while in candidature for a research degree at this University.
- Where I have consulted the published work of others, this is always clearly attributed.
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- I have acknowledged all main sources of help.

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Date:

"The scientist finds his rewards in what Henri Poincaré calls the joy of comprehension, and not in the possibilities of application to which any discovery may lead."

Albert Einstein

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Abstract

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by Ailier RIVERO ACOSTA

Spin-1 matter fields are relevant both in the description of hadronic states and in some constructions of theories Beyond the Standard Model. In this work we study the renormalization of a model based on a spin-1 matter Lagrangian formulated in terms of an antisymmetric field transforming in the $(1,0) \oplus (0,1)$ representation of the Homogeneous Lorentz Group. The model includes an arbitrary gyromagnetic factor and self-interactions of the spin-1 field, which has mass dimension one. We describe the properties of the model and the Feynman rules. It is shown that the considered model is renormalizable at one-loop order for any value of the gyromagnetic factor and the β functions are determined. We also found that there is no fixed points for any real value of g different from 0.

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"When you drink water,
think of its source."
Chinese Proverb

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Contents

Introduction

In the formulation of the Standard Model [1–3] only a few representations of the Lorentz group are used. In general, this is not a restriction to the use of high spin representations in theories of physics Beyond the Standard Model like Supersymmetry [4]. Some constructions for high spin fields are the Dirac-Fierz-Pauli equations [5, 6] for half-integral spin particles and the Rarita–Schwinger [7] equation which is the relativistic field equation of spin-3/2 fermions.

Specifically, the $(1,0) \oplus (0,1)$ representation is used in [8] to describe low-energy interactions of the low-lying nonets of vector and axial-vector mesons [9]. Moreover, in Physics Beyond the Standard Model spin-one matter particles described by tensor fields have been proposed in [10]. According to [11], spin-1 particles are described in this representation by an antisymmetric field tensor of second rank.

The goal of the present work is to study the renormalization properties of a model which describes a spin-1 matter field using techniques described in [12–21] to calculate the radiative corrections at one-loop order. Specifically, we use a Lagrangian whose kinetic part is of Klein-Gordon type and whose spin-1 information is encoded by a Pauli-like term modulated by an arbitrary gyromagnetic factor g, which couples the matter field with the photons through the Lorentz generators. We also aim to find the β functions associated with the parameters involved in the Lagrangian and to observe its behavior to different energy values. This research is a direct generalization of the spin-1/2 case [22, 23].

The difference between the pure spin-1 representation $(1,0) \oplus (0,1)$, described by an antisymmetric tensor field of second rank, and the more familiar $(1/2,1/2)$ vector field is more dramatic in the massless case, as the Kalb-Ramond antisymmetric gauge field contains only one physical longitudinal degree of freedom [24], whereas the massless vector gauge field is characterized by 2 transverse ones. Switching to massive spin-1 particles, one must distinguish between gauge invariant and non-gauge invariant theories. It can be shown that a massive Stueckelberg compensated Kalb-Ramond gauge field is dual to a compensated massive gauge vector field [25]. However, for non-gauge invariant massive spin-1 theories, the properties of four-vector and antisymmetric tensor particles can differ significantly. In [10] the difference between spin-1 antisymmetric tensor mesons and the four-vector mesons has been studied in detail for composite hadrons. In the present work, we focus instead on pointlike massive spin-1 bosons, with emphasis on their electromagnetic properties and their possible self-interactions.

The model studied here is based on [11], where the complex antisymmetric tensor field has 6 complex degrees of freedom, making the $(1,0) \oplus (0,1)$ theory explicitly different to any of a massive gauge vector field. In [11] the Compton scattering of spin-1 particles described by both a massive four-vector and an antisymmetric tensor was analyzed for arbitrary values of the gyromagnetic factor, finding that the Compton scattering cross section off the parity degrees of freedom in $(1,0) \oplus (0,1)$ is finite in the forward direction, though it is still divergent elsewhere. Interestingly, for the antisymmetric tensor this result is independent of the gyromagnetic factor, while Compton scattering off the four-vector is only well behaved in all directions provided the gyromagnetic ratio is set to $g = 2$. Given the non-finiteness of Compton scattering in this theory, it is unclear if the renormalizable theory described here corresponds to a perturbation theory about a sensible zeroth-order Hamiltonian. However, it constitutes a unique theoretical laboratory from the point of view of the renormalization group, in the same spirit as scalar $\lambda\phi^3$ theory.

In nature there are several theories that describe with precision some phenomena but they are not renormalizable. This means that they only describe physical phenomena occurring at a determined length scale or energy scale. These kinds of theories are considered effective theories. Effective Field Theories are widely used in Particle Physics, Condensed Matter Physics, General Relativity and other branches of physics. Some examples of these ones are the Fermi theory of beta decay, chiral perturbation theory, a low-energy effective theory of the Standard Model known as Standard Model Effective Field Theory (SMEFT) [20, 26, 27] and even General Relativity itself is expected to be an effective field theory of a full theory of quantum gravity. One of the most common effective theories of general relativity is the Non-Relativistic General Relativity [28, 29].

Renormalization is a very important procedure in Quantum Field Theory (QFT). This one makes the theories to be predictive because of the fact that allows us to describe the behavior of a system at any possible energy scale. Actually, the renormalization group and the analysis of symmetries are useful when constructing effective field theories. A description of this technique for several theories like Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD) can be consulted in [16–23].

The analysis of our model will allow us to acquire a better understanding of the renormalization process and the implications that could have whether the theory is renormalizable or not for different values of the gyromagnetic factor. The theory we are working with is not physical but yet it is interesting to study the renormalization of this one. This research will show us if the construction of pure QED with matter spin-1 particles without the inclusion of self-interactions is possible and we also want to confirm that our model is gauge invariant and that it is symmetric under charge conjugation.

The signature that has been adopted in the metric is $(+,-,-,-)$ and natural units (speed of light c = 1, reduced Planck's constant $\hbar = 1$) are used throughout the text. The rest of the thesis is organized as follows.

The Introduction gives a brief revision about the subject of our work. Chapter 1 sets the theoretical fundamentals that are useful to understand the rest of the thesis and gives a small description of the work previously carried out in [22] for spin 1/2. Chapter 2 presents the model we are going to investigate and the determination of the Feynman rules for this one. In

Chapter 3 it is presented a first study of the renormalization of a simplified model considering only the self-interacting terms. Chapter 4 exposes the study of the Quantum Electrodynamics of the complete model, the determination of the β functions and the fixed points found for the model. The Conclusions sums up the main and general results obtained after completing the research. In the Appendix A is presented the general procedure to obtain the Feynman rules and the Appendix B describes the steps to compute the beta functions and anomalous dimensions of the model.

Chapter 1

Theoretical Fundamentals

In this Chapter we present a short summary of the work previously carried out in [22] for spin 1/2. We also introduce some of the theory fundamentals that will be necessary to understand the rest of the thesis in order to make this document more self-contained.

1.1 Special Relativity

In 1905 Albert Einstein published a very important article [30] that meant the birth of a theory that changed radically some ideas of that time: Special Relativity.

The two main postulates of this theory state that [30–34]:

- The laws of Physics are the same for all the inertial reference frames.
- The speed of light in the vacuum is constant in any inertial reference frame regardless of the motion of the source.

One implication of the theory is that the interval, defined as

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2, \tag{1.1}$$

is invariant under the Lorentz transformations. These transformations for two reference frames where the frame S is at rest and the frame S' is moving at a speed v with respect to the first one are given by

$$\begin{aligned} t' &= \gamma(t + vx), \\ x' &= \gamma(x + vt), \\ y' &= y, \\ z' &= z, \end{aligned} \tag{1.2}$$

where $\gamma = (1 - \beta^2)^{-1/2}$ and $\beta = v$. And the interval remains invariant $ds'^2 = ds^2$ as it was stated above.

Now the mathematical formalism of Special Relativity will be presented. A more detailed description can be found in [12, 35–37].

In theoretical physics, if the form of the physical laws that describe a theory remains invariant under arbitrary differentiable coordinate transformations it is said that they are covariant. In this case, it is required that quantities should be Lorentz covariant, then they transform appropriately under the elements of the Lorentz group. To achieve this, it is necessary to write the theory in terms of certain well-defined mathematical objects, such as scalars, vectors and tensors.

Scalar objects are invariant under rotations and boosts, like the mass or interval ds^2, among other quantities. The prototypical vector is expressed as

$$a^\mu = (a^0, a^1, a^2, a^3). \tag{1.3}$$

In the case of the space-time this is

$$x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z) = (t, \mathbf{x}). \tag{1.4}$$

There are two inequivalent types of vectors in Minkowski space which are denoted as contravariant and covariant. The first kind of vector transforms in the same way as dx^μ does:

$$dx'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} dx^\nu, \tag{1.5}$$

then

$$a'^\mu \equiv \frac{\partial x'^\mu}{\partial x^\nu} a^\nu = \Lambda^\mu{}_\nu a^\nu. \tag{1.6}$$

The second kind transforms as the gradient

$$\frac{\partial \phi}{\partial x'^\mu} = \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial \phi}{\partial x^\nu}, \tag{1.7}$$

therefore

$$b'_\mu \equiv \frac{\partial x^\nu}{\partial x'^\mu} b_\nu = \Lambda_\mu{}^\nu b_\nu. \tag{1.8}$$

Here $\Lambda_\mu{}^\nu$ is the inverse of $\Lambda^\mu{}_\nu$. In particular, the Lorentz transformation in eq.(1.2) $x'^\mu = \Lambda^\mu{}_\nu x^\nu$ can be written in matrix form as follows

$$\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \tag{1.9}$$

with the parameter ϕ, known as rapidity, defined as $\tanh\phi = \beta$.

There exists an object known as the metric, that can be used to write down the coefficients of the differentials in the interval, which in this case are just ± 1. The metric of Special Relativity is given by

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.10}$$

The metric has an inverse, which in this case is exactly the same matrix and it is denoted with upper indexes

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.11}$$

The metric and its inverse satisfy the relation

$$g^{\mu\nu} g_{\nu\rho} = \delta^{\mu}_{\rho}. \tag{1.12}$$

In terms of the metric tensor, the invariant interval reads

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}. \tag{1.13}$$

Another important fact about the metric is that can be used to raise or lower an index in the following way

$$x_{\mu} \equiv g_{\mu\nu} x^{\nu}, \qquad x^{\mu} = g^{\mu\nu} x_{\nu}. \tag{1.14}$$

It is also important to write the useful relation

$$g^{\rho\mu} \Lambda_{\mu}{}^{\nu} g_{\nu\sigma} = \Lambda^{\rho}{}_{\sigma}, \tag{1.15}$$

which can be clearly deduced from

$$b'^{\rho} = g^{\rho\mu} b'_{\mu} = g^{\rho\mu} \Lambda_{\mu}{}^{\nu} g_{\nu\sigma} b^{\sigma} = \Lambda^{\rho}{}_{\sigma} b^{\sigma}. \tag{1.16}$$

For consistency, the relation

$$ds'^2 = g_{\mu\nu} dx'^{\mu} dx'^{\nu} = g_{\mu\nu} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} dx^{\rho} dx^{\sigma} = g_{\rho\sigma} dx^{\rho} dx^{\sigma} = ds^2, \tag{1.17}$$

shows the invariance of the interval. From the previous expression can be determined the relation

$$\Lambda^{\mu}{}_{\rho} g_{\mu\nu} \Lambda^{\nu}{}_{\sigma} = g_{\rho\sigma}. \tag{1.18}$$

Some examples of four-vectors are:

- The energy-momentum four-vector $p^{\mu} = (E, \mathbf{p})$, where E is the energy and $\mathbf{p}$ is the momentum.

- The current density four-vector $j^\mu = (\rho, \mathbf{j})$, where ρ is the charge density and $\mathbf{j}$ is the current density.
- The vector potential four-vector $A^\mu = (\varphi, \mathbf{A})$, where φ is the scalar electric potential and $\mathbf{A}$ is the magnetic vector potential.
- The four-dimensional derivative operator ∂_μ is also a combination of a time-like part and a space-like part, and is defined by

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \equiv \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right) \equiv (\partial_0, \partial_1, \partial_2, \partial_3) \equiv (\partial_0, \nabla). \tag{1.19}$$

Here is important to note that the lower index written in ∂_μ is in contrast with the upper index on four-vectors like x^μ. This derivative satisfies the following

$$\partial_\mu x^\nu = \frac{\partial x^\nu}{\partial x^\mu} = \delta^\nu_\mu. \tag{1.20}$$

The derivative operator with upper indices is then defined as

$$\partial^\mu \equiv g^{\mu\nu} \partial_\nu = (\partial_0, -\nabla). \tag{1.21}$$

Thus, the D'Alembertian operator is given by

$$\Box \equiv \partial^2 \equiv \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2. \tag{1.22}$$

Using relativistic notation, the conservation of charge can be written simply as

$$\partial_\mu j^\mu = \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \tag{1.23}$$

1.2 Lorentz Group

The Lorentz group is a group of four-by-four matrices performing Lorentz transformations on the four-dimensional Minkowski space. The transformations leave invariant the interval $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$. The generators of this group $SO(1,3)$ are $\mathrm{M}_{\mu\nu}$, $(\mu, \nu = 0, 1, 2, 3)$. These generators satisfy the following algebra in 4 dimensions [4, 22, 37–39]

$$[\mathrm{M}_{\mu\nu}, \mathrm{M}_{\rho\sigma}] = i(g_{\mu\rho}\mathrm{M}_{\nu\sigma} - g_{\nu\rho}\mathrm{M}_{\mu\sigma} + g_{\nu\sigma}\mathrm{M}_{\mu\rho} + g_{\mu\sigma}\mathrm{M}_{\nu\rho}). \tag{1.24}$$

Then, any element of the Lorentz group can be written in general as

$$\Lambda = e^{-\frac{i}{2}\Omega_{\mu\nu}\cdot \mathbf{M}^{\mu\nu}}, \tag{1.25}$$

where $\Omega_{\mu\nu}$ is a matrix of real parameters that includes the angles of the rotations and boosts (θ_i and ϕ_i).

It is customary to separate the Lorentz generators as presented previously in generators of rotations **J** and boosts generators **K**. They are related to $M_{\mu\nu}$ as follows

$$J_i \equiv \frac{1}{2}\epsilon_{ijk}M^{jk}, \qquad K_i \equiv M^{0i}. \tag{1.26}$$

There are a total of six generators: three for rotations and three for boosts. Thus, the Lorentz group is a six-parameters group.

The generators for the proper rotations are J_1, J_2, J_3 and they are given by

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1.27}$$

For example, the rotation along the x-axis can be written as

$$\Lambda_1 = e^{-i\theta_1 J_1}, \tag{1.28}$$

or in matrix form

$$\Lambda_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}. \tag{1.29}$$

The Boosts generators are K_1, K_2, K_3 and they are given by

$$K_1 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}. \tag{1.30}$$

For example, the boost in the t-x plane can be written as

$$\Lambda_{t-x} = e^{-i\phi_1 K_1}, \tag{1.31}$$

or in matrix form

$$\Lambda_{t-x} = \begin{pmatrix} \cosh\phi_1 & \sinh\phi_1 & 0 & 0 \\ \sinh\phi_1 & \cosh\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1.32}$$

The 6 generators of the Lorentz Group satisfy the algebra 1.24 which can be explicitly written for the J and K [40–42] as

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \qquad [J_i, K_j] = i\epsilon_{ijk}K_k, \qquad [K_i, K_j] = -i\epsilon_{ijk}J_k. \tag{1.33}$$

There exists also a relation between the general parameter of the Lorentz transformations $\Omega_{\mu\nu}$ and the parameters of the rotations and boosts (θ_i and ϕ_i):

$$\theta_i \equiv \frac{1}{2}\epsilon_{ijk}\Omega_{jk}, \qquad \phi_i \equiv \Omega_{0i}. \tag{1.34}$$

In general, the elements of this group can also be written as follows

$$\Lambda = e^{-i(\Theta\cdot\mathbf{J}+\Phi\cdot\mathbf{K})}, \tag{1.35}$$

which is equivalent to eq.(1.25):

$$\Lambda = e^{-\frac{i}{2}\Omega_{\mu\nu}\cdot\mathbf{M}^{\mu\nu}} = e^{-i(\Theta\cdot\mathbf{J}+\Phi\cdot\mathbf{K})}. \tag{1.36}$$

The Lorentz Group ($SO(1,3)$) is isomorphic to the group $SU(2)_A \otimes SU(2)_B$ generated by

$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}), \qquad \mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}). \tag{1.37}$$

These generators satisfy the commutation relations

$$[A_i, A_j] = i\epsilon_{ijk}A_k, \qquad [B_i, B_j] = i\epsilon_{ijk}B_k, \qquad [A_i, B_j] = 0. \tag{1.38}$$

Some of the most common representations of the Lorentz group are:

- $(0,0)$ is the Lorentz scalar representation. This representation is carried by relativistic scalar field theories.
- $\left(\frac{1}{2},0\right)$ is the left-handed Weyl spinor and $\left(0,\frac{1}{2}\right)$ is the right-handed Weyl spinor representation.
- $\left(\frac{1}{2},\frac{1}{2}\right)$ is the four-vector representation. The four-momentum of a particle transforms under this representation.
- $(1,0)$ is the self-dual tensor and $(0,1)$ is the anti-self-dual tensor.
- $(1,1)$ is the spin 2 representation of a traceless symmetric tensor field [43].

Among the irreducible representations (irreps.) of the Lorentz Group there are only a few that are used to describe the particles. In fact, the Standard Model is constructed only using the representations: $(0,0)$ for the Higgs Boson; $\left(\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2}\right)$ for the quarks and leptons and $\left(\frac{1}{2},\frac{1}{2}\right)$ for the gauge bosons[39].

For the representation $\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)$ the authors in [4] construct a parity-based covariant basis given by

$$\{1, \chi, S_{\mu}, \chi S_{\mu}, M_{\mu\nu}\}\,, \tag{1.39}$$

their procedure reproduces the conventional covariant basis with the γ^{μ} matrices in the Weyl representation

$$\{1, \gamma_5, \gamma^{\mu}, \gamma_5\gamma^{\mu}, \sigma_{\mu\nu}\}\,. \tag{1.40}$$

The parity-based covariant basis for a general $(j,0) \oplus (0,j)$ operator space, according to [4, 9], contains the following set of elements:

- The unit matrix of dimension $2(2j+1)$ and the chirality operator χ. These are Lorentz scalar operators.
- Six operators transforming in the $(1,0) \oplus (0,1)$ representation. These ones form a rank-2 antisymmetric tensor, $\mathrm{M}_{\mu\nu}$, whose components are the generators of the HLG.
- A pair of symmetric traceless matrix tensors, denoted by $S^{\mu_1\mu_2\cdots\mu_{2j}}$ and $\chi S^{\mu_1\mu_2\cdots\mu_{2j}}$, transforming in the (j,j) representation.
- A series of tensor matrix operators with the appropriate symmetry properties such that they transform in the $(2,0) \oplus (0,2)$;$(3,0) \oplus (0,3)$; $(2j,0) \oplus (0,2j)$ representations of the HLG.

In our work, we are going to use the representation $(1,0) \oplus (0,1)$. Then, with the ingredients described previously, the basis of matrices with well-defined Lorentz transformation properties [4, 9, 39] for our case is given by

$$\left\{1, \chi, S^{\mu\nu}, \chi S^{\mu\nu}, M^{\mu\nu}, C^{\mu\nu\alpha\beta}\right\}, \tag{1.41}$$

where the elements are defined as follows [4, 9, 11, 39, 44]:

$$1_{\alpha\beta\gamma\delta} = \frac{1}{2}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}), \qquad \chi_{\alpha\beta\gamma\delta} = \frac{i}{2}\epsilon_{\alpha\beta\gamma\delta}, \tag{1.42}$$

$$(M_{\mu\nu})_{\alpha\beta\gamma\delta} = -i(g_{\mu\gamma}1_{\alpha\beta\nu\delta} + g_{\mu\delta}1_{\alpha\beta\gamma\nu} - g_{\gamma\nu}1_{\alpha\beta\mu\delta} - g_{\delta\nu}1_{\alpha\beta\gamma\mu}), \tag{1.43}$$

$S^{\mu\nu}$ is a symmetric traceless tensor ($S^{\mu}_{\ \mu} = 0$), defined as

$$(S_{\mu\nu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - g_{\mu\gamma}1_{\alpha\beta\nu\delta} - g_{\mu\delta}1_{\alpha\beta\gamma\nu} - g_{\gamma\nu}1_{\alpha\beta\mu\delta} - g_{\delta\nu}1_{\alpha\beta\gamma\mu}. \tag{1.44}$$

The tensor $C^{\mu\nu\alpha\beta}$ is defined as

$$C^{\mu\nu\alpha\beta} = 4\left\{M^{\mu\nu}, M^{\alpha\beta}\right\} + 2\left\{M^{\mu\alpha}, M^{\nu\beta}\right\} - 2\left\{M^{\mu\beta}, M^{\nu\alpha}\right\} - 16(1^{\mu\nu\alpha\beta}). \tag{1.45}$$

$S^{\mu\nu}$ satisfies the relations

$$[S^{\mu\nu}, S^{\alpha\beta}] = -i(g^{\mu\alpha}M^{\nu\beta} + g^{\nu\alpha}M^{\mu\beta} + g^{\nu\beta}M^{\mu\alpha} + g^{\mu\beta}M^{\nu\alpha}), \tag{1.46}$$

$$\left\{S^{\mu\nu}, S^{\alpha\beta}\right\} = \frac{4}{3}\left(g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right) - \frac{1}{6}(C^{\mu\alpha\nu\beta} + C^{\mu\beta\nu\alpha}). \tag{1.47}$$

$C^{\mu\nu\alpha\beta}$ is a traceless tensor, which means that the contraction of any pair of indices vanishes. It also satisfy the Bianchi identity

$$C^{\mu\nu\alpha\beta} + C^{\mu\alpha\beta\nu} + C^{\mu\beta\nu\alpha} = 0, \tag{1.48}$$

and it has the symmetries

$$C^{\mu\nu\alpha\beta} = -C^{\nu\mu\alpha\beta} = -C^{\mu\nu\beta\alpha}, \qquad C^{\mu\nu\alpha\beta} = C^{\alpha\beta\mu\nu}. \tag{1.49}$$

These symmetries allow to reduce the 256 components of a general four-index tensor to only 10 independent components.

Defining $\widetilde{C}^{\mu\rho\nu\sigma} \equiv \frac{1}{2}\epsilon^{\mu\rho}{}_{\alpha\beta}C^{\alpha\beta\nu\sigma} = -i\chi C^{\mu\rho\nu\sigma}$ and $\tilde{M}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}M_{\alpha\beta}/2$, we also have for the covariant basis of eq.(1.41) the commutation rules

$$i[M^{\mu\nu}, S^{\rho\sigma}] = g^{\mu\rho}S^{\nu\sigma} - g^{\nu\rho}S^{\mu\sigma} + g^{\mu\sigma}S^{\nu\rho} - g^{\nu\sigma}S^{\mu\rho}, \tag{1.50}$$

$$[\chi S^{\mu\nu}, \chi S^{\rho\sigma}] = ig^{\mu\rho}M^{\nu\sigma} + ig^{\nu\rho}M^{\mu\sigma} + ig^{\mu\sigma}M^{\nu\rho} + ig^{\nu\sigma}M^{\mu\rho}, \tag{1.51}$$

$$[\chi S^{\mu\nu}, S^{\rho\sigma}] = \frac{4}{3}\left(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - \frac{1}{2}g^{\mu\nu}g^{\rho\sigma}\right) - \frac{i}{6}\left(\widetilde{C}^{\mu\rho\nu\sigma} + \widetilde{C}^{\mu\sigma\nu\rho}\right), \tag{1.52}$$

$$[\chi, S^{\mu\nu}] = 2\chi S^{\mu\nu}, \tag{1.53}$$

$$[\chi, \chi S^{\mu\nu}] = 2S^{\mu\nu}, \tag{1.54}$$

and the anticommutators

$$\{M^{\mu\nu}, S^{\rho\sigma}\} = \epsilon^{\mu\nu\alpha\sigma}\chi S^{\rho}{}_{\alpha} + \epsilon^{\mu\nu\alpha\rho}\chi S^{\sigma}{}_{\alpha}, \tag{1.55}$$

$$\{\chi S^{\mu\nu}, \chi S^{\rho\sigma}\} = -\frac{4}{3}\left(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - \frac{1}{2}g^{\mu\nu}g^{\rho\sigma}\right) + \frac{1}{6}\left(C^{\mu\rho\nu\sigma} + C^{\mu\sigma\nu\rho}\right), \tag{1.56}$$

$$\{\chi S^{\mu\nu}, S^{\rho\sigma}\} = \frac{1}{2}\left(g^{\mu\rho}\widetilde{M}^{\nu\sigma} + g^{\nu\sigma}\widetilde{M}^{\mu\rho}\right) + \frac{1}{2}\left(g^{\mu\sigma}\widetilde{M}^{\nu\rho} + g^{\nu\rho}\widetilde{M}^{\mu\sigma}\right), \tag{1.57}$$

$$\{M^{\mu\nu}, M^{\rho\sigma}\} = \frac{4}{3}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) - \frac{8}{6}i\epsilon^{\mu\nu\rho\sigma}\chi + \frac{1}{6}C^{\mu\nu\rho\sigma}, \tag{1.58}$$

$$\{\chi, S^{\mu\nu}\} = 0, \tag{1.59}$$

$$\{\chi, \chi S^{\mu\nu}\} = 0. \tag{1.60}$$

1.3 Some Aspects of Quantum Field Theory

In the development of Quantum Field Theory [12–15, 35, 45–48] during the last century there were developed techniques that made it a really consistent and successful theory. Some of these are the Dimensional Regularization and the Renormalization.

Dimensional regularization and renormalization are very powerful tools used in modern theoretical physics[12, 13, 16, 18]. Dimensional regularization was introduced by Carlos Bollini and

Juan José Giambiagi[52] in 1972 and simultaneously by Gerardus't Hooft and Martinus J. G. Veltman in 1972-1973 [49, 50]. Precisely this work made t Hooft and Veltman receive the Nobel Prize in Physics in 1999 [51] for the application of this method to non-abelian electroweak theory.

1.3.1 Dimensional Regularization

Dimensional regularization has the important advantage with respect to other regularization schemes, like cutoff regularization, that it respects gauge and Lorentz symmetries [16].

The key idea of dimensional regularization is to change the number of dimensions of the problem considered and this is achieved computing the volume $V(x)$ in n-dimensions. Here n does not have to be necessarily an integer [16, 49, 50, 52].

For example, for an infinite charged line [1] the potential is given by the expression

$$\phi(x) = \frac{\lambda}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{x^2 + y^2}}. \tag{1.61}$$

Using dimensional regularization the integration along y is changed for integration along a general 1-dimension volume as follows

$$\int_{-\infty}^{+\infty} dy = \int dV_1. \tag{1.62}$$

In general, for n dimensions, the integration over the volume V_n is defined as

$$\int dV_n = \int d\Omega_n \int_0^{+\infty} y^{n-1} dy, \tag{1.63}$$

where Ω_n is given by

$$\Omega_n = \int d\Omega_n \quad = \quad \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} \equiv \frac{n\,\pi^{n/2}}{\Gamma\left(\frac{n}{2} + 1\right)} \quad . \tag{1.64}$$

Here, Ω_n is the solid-angle in n-dimensions, and we have used the property of the Gamma function: $\Gamma(z+1) = z\,\Gamma(z)$.

1.3.2 Renormalization

In Quantum Field Theory in general infinities arise in important quantities that need to be compared with experimental measurements [19]. There have been some works that presented a way of describing these apparently divergent theories in a consistent manner [54–58].

Renormalization consists of removing these infinite parts in such a way that the considered theory remains consistent. In general a renormalization scheme has two components[18]. First

[1] For a more detailed and explicit development of this example, see [16, 53]

it has to be applied the process known as regularization. This step isolates the infinities appearing in the Feynman diagrams. The regularization is arbitrary, the only requirement is that it must respect the symmetries of the theory.

After the regularization, it is necessary to apply a systematic method to remove the divergences. This process is called renormalization scheme. The selection of the subtraction method that leads to the renormalized theory is also arbitrary. Here the important fact to be considered is that the physical results should not depend on this choice [17–21].

In general, the procedure consists in defining a relation between the parameters of the original Lagrangian (commonly called "bare Lagrangian") and the ones of the renormalized Lagrangian. This is justified because when observing at very short distances of particles, there are other factors to consider. For example, in an electron theory a first description states an initial mass and charge, but in QFT a cloud of virtual particles, such as photons, positrons, and others surrounds and interacts with the initial electron. Then, the electron at such distances has a slightly different mass and electric charge than does the dressed electron seen at large distances.

Mathematically, the process is to replace the initially postulated parameters of the theory with the renormalized ones, which are the experimentally observed values. The parameters are related through some constants that are absorbing the divergent quantities that arise in the calculations. A detailed development of this procedure for several theories like QED and Quantum Chromodynamics (QCD) can be consulted in references [16–21, 23].

1.4 Case for self-interacting second order spin 1/2 fermions

In [22] the authors study the "Renormalization of the QED of self-interacting second order spin 1/2 fermion". The Lagrangian considered in this paper is

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D^{\dagger\mu}\bar{\psi}T_{\mu\nu}D^{\nu}\psi - m^2\bar{\psi}\psi + \frac{\lambda_1}{2}\left(\bar{\psi}\psi\right)^2 + \frac{\lambda_2}{2}\left(\bar{\psi}\gamma^5\psi\right)\left(\bar{\psi}\gamma^5\psi\right) \\ &+ \frac{\lambda_3}{2}\left(\bar{\psi}M^{\mu\nu}\psi\right)\left(\bar{\psi}M_{\mu\nu}\psi\right),\end{aligned} \tag{1.65}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the gauge covariant derivative, A^μ is the gauge field, ψ is the fermion field, the fermion charge is $-e$, λ_j $(j = 1, 2, 3)$ are the couplings of the three possible dimension four fermion self-interaction terms. This Lagrangian has a gauge symmetry. The space-time tensor $T^{\mu\nu}$ is defined as

$$T^{\mu\nu} \equiv g^{\mu\nu} - igM^{\mu\nu} - ig'\tilde{M}^{\mu\nu}. \tag{1.66}$$

To renormalize the theory it is necessary to introduce some counterterms. These counterterms are going to absorb the divergent part of the amplitudes. Then, the parameters of the bare Lagrangian [2] are the fermion mass m_0, the fermion charge e_0, the gyromagnetic factor g_0 and

[2]The bare Lagrangian $\mathcal{L}_0$ is exactly as 1.65 only including a subscript 0 to every parameter and field.

the self-interaction couplings λ_{0j} ($j = 1, 2, 3$). The renormalized fields are related to the bare ones as

$$A_r^\mu = Z_1^{-\frac{1}{2}} A_0^\mu, \qquad \psi_r = Z_2^{-\frac{1}{2}} \psi_0. \tag{1.67}$$

After splitting the Lagrangian in a convenient way and using the definitions:

$$\begin{aligned}
&\delta_1 \equiv Z_1 - 1, \qquad \delta_2 \equiv Z_2 - 1, \qquad \delta_m \equiv Z_m - Z_2, \qquad Z_m \equiv \frac{m_0^2}{m_r^2} Z_2, \\
&\delta_e \equiv Z_e - 1, \quad \delta_3 \equiv Z_3 - 1, \quad \delta_{\lambda_j} \equiv Z_{\lambda_j} - 1, \quad \delta_g \equiv Z_{eg} - Z_e, \\
&Z_e \equiv \frac{e_0}{e_r} Z_1^{\frac{1}{2}} Z_2, \quad Z_3 \equiv \frac{e_0^2}{e_r^2} Z_1 Z_2, \quad Z_{\lambda_j} \equiv \frac{\lambda_{0j}}{\lambda_{rj}} Z_2^2, \qquad Z_{eg} \equiv \frac{g_0}{g_r} Z_e,
\end{aligned} \tag{1.68}$$

the following expressions for the counterterms are obtained

$$\delta_1 = -\frac{e^2 \tau}{(4\pi)^2} \left(\frac{g^2}{4} - \frac{1}{3} \right) \left[\frac{1}{\tilde{\epsilon}} - \ln \frac{m^2}{\mu^2} \right], \tag{1.69}$$

$$\delta_m = \frac{1}{(4\pi)^2} \left\{ \left[\frac{1}{\tilde{\epsilon}} - \ln \frac{m^2}{\mu^2} \right] \left[(\tau - 1)\lambda_1 - \lambda_2 - 3\lambda_3 - 3\left(1 + \frac{g^2}{4}\right) e^2 \right] \right. \tag{1.70}$$

$$\left. + (\tau - 1)\lambda_1 - \lambda_2 + \frac{\lambda_3}{2} - \left(7 + \frac{g^2}{4}\right) e^2 \right\}, \tag{1.71}$$

$$\delta_2 = \delta_e = \delta_3 = \frac{e^2}{(4\pi)^2} (3 - \xi) \left[\frac{1}{\tilde{\epsilon}} - \ln \frac{m^2}{\mu^2} - \ln \frac{m_\gamma^2}{m^2} \right], \tag{1.72}$$

$$\delta_g = -\frac{1}{(4\pi)^2} \left(\frac{1}{\tilde{\epsilon}} - \ln \frac{m^2}{\mu^2} \right) \left[\left(1 - \frac{g^2}{4}\right) e^2 + \lambda_1 + \lambda_2 - \left(1 + \frac{\tau}{2}\right) \lambda_3 \right], \tag{1.73}$$

$$\delta_{\lambda_1} = -\frac{1}{(4\pi)^2} \frac{1}{\tilde{\epsilon}} \left\{ 6 \left(1 - \frac{g^2}{4}\right)^2 \frac{e^4}{\lambda_1} + e^2 \left[\frac{3g^2}{2} \left(1 + \frac{\lambda_3}{\lambda_1}\right) + 2\xi \right] \right.$$

$$\left. + (4 - \tau)\lambda_1 + 2\frac{\lambda_2^2}{\lambda_1} + 3\frac{\lambda_3^2}{\lambda_1} + 2\lambda_2 + 6\lambda_3 \right\}, \tag{1.74}$$

$$\delta_{\lambda_2} = -\frac{1}{(4\pi)^2} \frac{1}{\tilde{\epsilon}} \left\{ e^2 \left[\frac{3g^2}{2} \left(1 + \frac{\lambda_3}{\lambda_2}\right) + 2\xi \right] + 3\frac{\lambda_3^2}{\lambda_2} \right.$$

$$\left. + 6\lambda_1 + (2 - \tau)\lambda_2 + 6\lambda_3 \right\}, \tag{1.75}$$

$$\delta_{\lambda_3} = -\frac{1}{(4\pi)^2} \frac{1}{\tilde{\epsilon}} \left\{ e^2 \left[\frac{g^2}{2} \left(2\frac{\lambda_1}{\lambda_3} + 2\frac{\lambda_2}{\lambda_3} - 1\right) + 2\xi \right] \right.$$

$$\left. + 6\lambda_1 + 6\lambda_2 - \left(\frac{4 + \tau}{2}\right) \lambda_3 \right\}. \tag{1.76}$$

The beta functions encode the dependence of the coupling parameters on the energy scale μ (also called the "running of the coupling") of a given physical process. This functions $\beta_\eta \equiv \mu \frac{\partial \eta}{\partial \mu}$ and the anomalous dimensions $\gamma_m \equiv \frac{\mu}{m} \frac{\partial m}{\partial \mu}$ in the $\epsilon \to 0$ limit are obtained in this paper as:

$$\beta_e = \frac{e^3 \tau}{48\pi^2} \left(\frac{3}{4} g^2 - 1 \right), \tag{1.77}$$

$$\beta_g = \frac{g}{32\pi^2}\left[e^2\left(g^2-4\right) - 4(\lambda_1+\lambda_2) + 4\left(1+\frac{\tau}{2}\right)\lambda_3\right], \tag{1.78}$$

$$\beta_{\lambda_1} = -\frac{1}{16\pi^2}\left\{\frac{3}{4}e^4\left(g^2-4\right)^2 + 3e^2\left[\left(4+g^2\right)\lambda_1 + g^2\lambda_3\right]\right. \tag{1.79}$$
$$\left. +2(4-\tau)\lambda_1^2 + 4\lambda_2\left(\lambda_1+\lambda_2\right) + 6\lambda_3\left(2\lambda_1+\lambda_3\right)\right\},$$

$$\beta_{\lambda_2} = -\frac{1}{16\pi^2}\left\{3e^2\left[\left(4+g^2\right)\lambda_2 + g^2\lambda_3\right]\right. \tag{1.80}$$
$$\left. +12\lambda_2\lambda_1 + 2(2-\tau)\lambda_2^2 + 6\lambda_3\left(2\lambda_2+\lambda_3\right)\right\},$$

$$\beta_{\lambda_3} = -\frac{1}{16\pi^2}\left\{e^2\left[\left(12-g^2\right)\lambda_3 + 2g^2\left(\lambda_1+\lambda_2\right)\right]\right. \tag{1.81}$$
$$\left. +12\lambda_3(\lambda_1+\lambda_2) - (4+\tau)\lambda_3^2\right\},$$

$$\gamma_m = \frac{1}{64\pi^2}\left\{-3e^2\left(g^2+4\right) + 4[(\tau-1)\lambda_1 - \lambda_2 - 3\lambda_3]\right\}. \tag{1.82}$$

In the cited work, the authors study the one-loop level renormalization of the electrodynamics of second order fermions in the Poincaré projector formalism, in an arbitrary covariant gauge and including fermion self-interactions. They proved that β_g vanishes for $g = \pm 2$, $\lambda_1 = \lambda_2 = \lambda_3 = 0$ showing that the theory has a simple connection to Dirac QED, as expected. They also showed that making the proper replacements in the expressions obtained for the beta functions they are able to recover the well known Dirac-QED beta function and the corresponding anomalous dimension of the mass

$$\beta_e^D = \frac{e^3}{12\pi^2}, \qquad \gamma_m^D = -\frac{3e^2}{8\pi^2}. \tag{1.83}$$

1.5 Ward-Takahashi Identities

In Quantum Field Theory, Ward–Takahashi identities are relations between correlation functions. They are the result of the global or gauge symmetries of the theory, and they remain valid after renormalization.

The first of these identities was originally used by John Clive Ward[60] and Yasushi Takahashi[61] in Quantum Electrodynamics to relate the wave function renormalization of the electron to its vertex renormalization factor. With this, they guaranteed the cancellation of the ultraviolet divergence to all orders in perturbation theory.

In general, a Ward–Takahashi identity is the quantum version of the classical current conservation that the Noether's theorem states that is related to a continuous symmetry. This generalized sense can be found well described in [13] and should not be confused with the specific case of the original Ward-Takahashi identity.

The Ward–Takahashi identity applies to correlation functions in momentum space and it is not a requirement that all their external momenta be on-shell. Let

$$\mathcal{M}(k; p_1 \dots p_n; q_1 \dots q_n) = \epsilon_\mu(k)\mathcal{M}^\mu(k; p_1 \dots p_n; q_1 \dots q_n), \tag{1.84}$$

be a QED correlation function. This function involves an external photon with momentum k, $\epsilon_\mu(k)$ is the polarization vector of the photon and the p_i are the momenta of the initial-state electrons, the q_i are the momenta of the final-state electrons. $\mathcal{M}_0$ is defined as the simpler amplitude that is obtained by removing the photon with momentum k from the original amplitude. Then the Ward–Takahashi identity reads

$$k_\mu \mathcal{M}^\mu(k; p_1 \cdots p_n; q_1 \cdots q_n) = e \sum_i \Big\{ \mathcal{M}_0(p_1 \cdots p_n; q_1 \cdots (q_i - k) \cdots q_n) - \mathcal{M}_0(p_1 \cdots (p_i + k) \cdots p_n; q_1 \cdots q_n) \Big\}, \tag{1.85}$$

where e is the charge of the electron.

For a more extensive exploration about this topic, can be consulted reference [18] for the case of non-abelian gauge theories, [62] for a derivation of the Ward–Takahashi identities for connected Green's functions in QED without using some techniques like equal-time commutation relations and the Feynman-Dyson perturbation expansions and [63] to see the violation of these identities in axial current anomalies [64, 65].

The gauge invariance of the theory imposes two important Ward-Takahashi identities (see [59] for their derivation in the analogous spin 1/2 case). The first one relates the tensor-tensor-photon (TTγ) vertex function $-ie\Gamma^\mu(q, p, -p-q)$, where q is the momentum of the photon, with the tensor self-energy $-i\Sigma(p)$ according to

$$\Gamma^\mu(0, p, -p) = -\frac{\partial \Sigma(p)}{\partial p_\mu}. \tag{1.86}$$

The second one involves the tensor-tensor-photon-photon (TT$\gamma\gamma$) vertex $ie^2\Gamma^{\mu\nu}(q, q', p, p')$, with photon momenta q and q', and the TTγ vertex, and reads

$$\Gamma^{\mu\nu}(0, q', p, p') = \frac{\partial \Gamma_\nu(q', p, p')}{\partial p_\mu} + \frac{\partial \Gamma_\nu(q', p, p')}{\partial p'_\mu}. \tag{1.87}$$

Chapter 2

Description of the model

In this Chapter we shall present the model we are considering and the Feynman rules obtained. The field considered is a massive complex spin-1 antisymmetric tensor $B^{\alpha\beta}$ in the $(1,0) \oplus (0,1)$ representation of the HLG. The kinetic part of the Lagrangian is of Klein-Gordon type. The spin-1 information is encoded by a Pauli-like term modulated by an arbitrary gyromagnetic factor g and the four independent quartic self-interactions that can be built from the covariant basis for this representation space, given by the complete set of tensors presented in eq.(1.41).

2.1 Lagrangian for the spin-1 field $B^{\alpha\beta}$

The Lagrangian of the spin-1 matter field $B^{\alpha\beta}$ is given by

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^{\mu}B^{\alpha\beta})^{\dagger}\,(T_{\mu\nu})_{\alpha\beta\gamma\delta}\,(D^{\nu}B^{\gamma\delta}) - m^2(B^{\alpha\beta})^{\dagger}B_{\alpha\beta} \\ & + \frac{\lambda_1}{2}(B^{\alpha\beta\,\dagger}1_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}1_{\mu\nu\rho\sigma}B^{\rho\sigma}) + \frac{\lambda_2}{2}(B^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma}B^{\rho\sigma}) \\ & + \frac{\lambda_3}{2}(B^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}) \\ & + \frac{\lambda_4}{2}(B^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}),\end{aligned} \tag{2.1}$$

where $D^{\mu} = \partial^{\mu} + ieA^{\mu}$ is the gauge covariant derivative, A^{μ} is the gauge field, $B^{\alpha\beta}$ is the boson field. This Lagrangian has a gauge symmetry. The tensors $F^{\mu\nu}$ and $(T_{\mu\nu})_{\alpha\beta\gamma\delta}$ are defined as

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}, \qquad (T_{\mu\nu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig(M_{\mu\nu})_{\alpha\beta\gamma\delta}. \tag{2.2}$$

In our analysis, the gauge freedom is fixed by the R_{ξ} contribution

$$\mathcal{L}_{\text{G.F.}} = -\frac{1}{2\xi}(\partial^{\mu}A_{\mu})^2. \tag{2.3}$$

With all the definitions presented in eqs.(2.2, 1.42-1.44) and with arbitrary gauge fixing parameter ξ, we can write the Lagrangian of the model in the following way

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^{\mu}A_{\mu})^2 + ((\partial^{\mu} + ieA^{\mu})B^{\alpha\beta})^{\dagger}\,(g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig(M_{\mu\nu})_{\alpha\beta\gamma\delta})\,(\partial^{\nu} + ieA^{\nu})B^{\gamma\delta}$$

$$-m^2(B^{\alpha\beta})^\dagger B_{\alpha\beta} + \frac{\lambda_1}{2}(B^{\alpha\beta\dagger}1_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\dagger}1_{\mu\nu\rho\sigma}B^{\rho\sigma}) + \frac{\lambda_2}{2}(B^{\alpha\beta\dagger}\chi_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\dagger}\chi_{\mu\nu\rho\sigma}B^{\rho\sigma})$$
$$+\frac{\lambda_3}{2}(B^{\alpha_1\beta_1\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2})$$
$$+\frac{\lambda_4}{2}(B^{\alpha_1\beta_1\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}), \quad (2.4)$$

and after some small transformations it can be written in a more suitable way

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \partial^\mu B^{\alpha\beta\dagger}\partial_\mu B_{\alpha\beta} - m^2(B^{\alpha\beta})^\dagger B_{\alpha\beta} \\ & -ieA^\mu[B^{\alpha\beta\dagger}(T_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B^{\gamma\delta} - (\partial^\nu B^{\alpha\beta\dagger})(T_{\nu\mu})_{\alpha\beta\gamma\delta}B^{\gamma\delta}] + e^2A^\mu A_\mu B^{\alpha\beta\dagger}B_{\alpha\beta} \\ & +\frac{\lambda_1}{2}(B^{\alpha\beta\dagger}1_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\dagger}1_{\mu\nu\rho\sigma}B^{\rho\sigma}) + \frac{\lambda_2}{2}(B^{\alpha\beta\dagger}\chi_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\dagger}\chi_{\mu\nu\rho\sigma}B^{\rho\sigma}) \\ & +\frac{\lambda_3}{2}(B^{\alpha_1\beta_1\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}) \\ & +\frac{\lambda_4}{2}(B^{\alpha_1\beta_1\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}). \end{aligned} \quad (2.5)$$

2.1.1 Determining the Feynman rules for our Lagrangian

In this section the Feynman rules for our model are going to be determined. The general procedure to determine the Feynman rules is described in Appendix A.

First term of the Lagrangian

Beginning with the first term of the Lagrangian which is

$$\Gamma_0 \supset \int d^4x[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2]. \quad (2.6)$$

Using the definition in eq.(2.2) for $F_{\mu\nu}$ we can write the first part of this term as

$$\Gamma_0 \supset \int d^4x[-\frac{1}{4}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)] = \int d^4x[-\frac{1}{2}F^{\mu\nu}\partial_\mu A_\nu], \quad (2.7)$$

and after a few steps

$$\begin{aligned}\Gamma_0 \supset & \int d^4x\frac{1}{2}[A^\nu g_{\nu\mu}\partial^\mu\partial_\mu A^\mu - A^\nu\partial^\mu\partial_\nu A_\mu] \\ = & \int d^4x\frac{1}{2}[A^\nu g_{\nu\mu}\partial^\mu\partial_\mu A^\mu - A^\nu\partial_\mu\partial_\nu A^\mu] \\ = & \frac{1}{2}\int d^4x A^\alpha[g_{\alpha\beta}\partial^\beta\partial_\beta - \partial_\alpha\partial_\beta]A^\beta. \end{aligned} \quad (2.8)$$

The second part of this term can be expressed as

$$-\frac{1}{2\xi}(\partial^\mu A_\mu)^2 = -\frac{1}{2\xi}\partial^\mu A_\mu\partial^\nu A_\nu = \frac{1}{2\xi}A_\mu\partial^\mu\partial^\nu A_\nu. \quad (2.9)$$

Then, we have

$$\Gamma_0 \supset \int d^4x \frac{1}{2} A^\alpha [g_{\alpha\beta}\partial^\beta\partial_\beta + (\frac{1}{\xi} - 1)\partial_\alpha\partial_\beta] A^\beta. \quad (2.10)$$

For this term the 2-points function reads

$$i\Gamma^2_{0\mu\nu}(p_1, p_2)(2\pi)^4\delta^4(p_1 + p_2) = i(2\pi)^8 \frac{\delta^2\Gamma_0[A^\alpha]}{\delta A^\mu(p_1)\delta A^\nu(p_2)}. \quad (2.11)$$

To continue with the procedure we need to use the Fourier transform, which is given by

$$A_\alpha(x) = \int \frac{d^4q_1}{(2\pi)^4} e^{-iq_1\cdot x} A_\alpha(q_1). \quad (2.12)$$

Then we can write the following expression

$$\begin{aligned}(2\pi)^8 i\Gamma_0 \quad \supset \quad & (2\pi)^8 i\frac{1}{2} \int d^4x \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} e^{-iq_1\cdot x} A^\alpha(q_1)[g_{\alpha\beta}\partial^\beta\partial_\beta \\ & + (\frac{1}{\xi} - 1)\partial_\alpha\partial_\beta] e^{-iq_2\cdot x} A^\beta(q_2). \quad (2.13)\end{aligned}$$

Now using that

$$\partial_\mu e^{-iq_2\cdot x} = e^{-iq_2\cdot x}(-iq_{2\nu}\partial_\mu x^\nu) = e^{-iq_2\cdot x}(-iq_{2\mu}), \quad (2.14)$$

we can write eq.(2.13) as

$$\begin{aligned}(2\pi)^8 i\Gamma_0 \quad \supset \quad & \frac{i}{2} \int d^4x d^4q_1 d^4q_2 e^{-i(q_1+q_2)\cdot x} A^\alpha(q_1)[g_{\alpha\beta}(-iq_2^\mu)(-iq_{2\mu}) \\ & + (\frac{1}{\xi} - 1)(-iq_{2\alpha})(-iq_{2\beta})] A^\beta(q_2). \quad (2.15)\end{aligned}$$

Then with the expression above, equation (2.11) becomes

$$\begin{aligned}i\Gamma^2_{0\,\mu\nu}(p_1, p_2)(2\pi)^4\delta^4(p_1 + p_2) \quad = \quad & -\frac{i}{2} \int d^4x d^4q_1 d^4q_2 e^{-i(q_1+q_2)\cdot x} [q_2^2 g_{\alpha\beta} \\ & + (\frac{1}{\xi} - 1) q_{2\alpha} q_{2\beta}] \frac{\delta A^\alpha(q_1)\delta A^\beta(q_2)}{\delta A^\mu(p_1)\delta A^\nu(p_2)}. \quad (2.16)\end{aligned}$$

Performing the functional derivatives and the integration respect to q_1 and q_2 we have

$$\begin{aligned}i\Gamma^2_{0\,\mu\nu}(p_1, p_2)(2\pi)^4\delta^4(p_1 + p_2) \quad = \quad & -\frac{i}{2} \int d^4x e^{-i(p_1+p_2)\cdot x} [p_1^2 g_{\nu\mu} \quad (2.17) \\ & + (\frac{1}{\xi} - 1) p_{1\nu} p_{1\mu} + p_2^2 g_{\mu\nu} + (\frac{1}{\xi} - 1) p_{2\mu} p_{2\nu}].\end{aligned}$$

And finally integrating respect to x

$$i\Gamma^2_{0\,\mu\nu}(p_1, p_2)(2\pi)^4\delta^4(p_1 + p_2) = -(2\pi)^4\delta^4(p_1 + p_2) 2\frac{i}{2} [p_1^2 g_{\mu\nu} + (\frac{1}{\xi} - 1) p_{1\nu} p_{1\mu}]. \quad (2.18)$$

With this expression and with $p_1 = p$ and $p_2 = -p$ we finally have

$$\Gamma^2_{0\mu\nu}(p,-p) = -p^2 g_{\mu\nu} - (\frac{1}{\xi} - 1) p_\mu p_\nu. \tag{2.19}$$

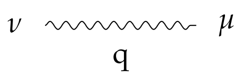
We need to find the propagator, which is the inverse of $\Gamma^2_{0\mu\nu}$ and can be obtained using the ansatz

$$[\Gamma^2_0]^{-1}_{\alpha\beta} = X g_{\mu\nu} + Y p_\alpha p_\beta. \tag{2.20}$$

Finally the propagator is given by

$$\Delta_{\mu\nu}(p) = i[\Gamma^2_0]^{-1}_{\mu\nu} = -\frac{i}{p^2}\left[g_{\mu\nu} + (\xi - 1)\frac{p_\mu p_\nu}{p^2}\right]. \tag{2.21}$$

And the Feynman diagram for this piece is represented as



Second term of the Lagrangian

Next, for the second term of the Lagrangian

$$\mathcal{L} \supset \partial^\mu B^{\rho\sigma\dagger}\partial_\mu B_{\rho\sigma} - m^2 (B^{\rho\sigma})^\dagger B_{\rho\sigma}, \tag{2.22}$$

the 2-points function reads

$$i\Gamma^2_{\alpha\beta\gamma\delta}(p_1,p_2)(2\pi)^4\delta^4(p_1+p_2) = i(2\pi)^8 \frac{\delta^2\Gamma_0}{\delta B_{\alpha\beta}(p_1)\delta B^\dagger_{\gamma\delta}(p_2)} \tag{2.23}$$
$$= i(2\pi)^8 \frac{\delta^2}{\delta B_{\alpha\beta}(p_1)\delta B^\dagger_{\gamma\delta}(p_2)}\left[\int d^4x \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}[\partial^\mu B^{\rho\sigma\dagger}\partial_\mu B_{\rho\sigma} - m^2(B^{\rho\sigma})^\dagger B_{\rho\sigma}]e^{-iq_1\cdot x}e^{-iq_2\cdot x}\right].$$

Now, using eq.(2.14) we can write

$$i\Gamma^2_{\alpha\beta\gamma\delta}(p_1,p_2)(2\pi)^4\delta^4(p_1+p_2) = i(2\pi)^8\frac{\delta^2}{\delta B_{\alpha\beta}(p_1)\delta B^\dagger_{\gamma\delta}(p_2)}$$
$$\times\left[\int d^4x \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}[(-iq_1^\mu)(-iq_{2\mu}) - m^2](B^{\rho\sigma})^\dagger B_{\rho\sigma}e^{-iq_1\cdot x}e^{-iq_2\cdot x}\right]. \tag{2.24}$$

Computing the functional derivatives

$$\begin{aligned}\frac{\delta^2[(B^{\rho\sigma}(q_1))^\dagger B_{\rho\sigma}(q_2)]}{\delta B_{\alpha\beta}(p_1)\delta B^\dagger_{\gamma\delta}(p_2)} &= \frac{\delta}{\delta B_{\alpha\beta}(p_1)}\left[\frac{\delta B^{\rho\sigma\dagger}(q_1)}{\delta B^\dagger_{\gamma\delta}(p_2)}\right]B_{\rho\sigma}(q_2)\\ &= g^{\rho\mu}g^{\sigma\nu}1^{\gamma\delta}_{\mu\nu}1^{\alpha\beta}_{\rho\sigma}\delta^4(q_1-p_2)\delta^4(q_2-p_1)\end{aligned}$$

$$= \quad 1^{\alpha\beta\gamma\delta}\delta^4(q_1-p_2)\delta^4(q_2-p_1). \tag{2.25}$$

Substituting this in eq.(2.24), this one can be written as

$$\begin{aligned} i\Gamma^2_{\alpha\beta\gamma\delta}(p_1,p_2)(2\pi)^4\delta^4(p_1+p_2) &= i\int d^4x d^4q_1 d^4q_2 e^{-i(q_1+q_2)\cdot x}[-q_1\cdot q_2-m^2] \\ &\times \; 1^{\alpha\beta\gamma\delta}\delta^4(q_1-p_2)\delta^4(q_2-p_1) \\ &= i[\int d^4x e^{-i(p_1+p_2)\cdot x}][-p_1\cdot p_2-m^2]1^{\alpha\beta\gamma\delta} \\ &= i[-p_1\cdot p_2-m^2](2\pi)^4\delta^4(p_1+p_2)1^{\alpha\beta\gamma\delta}. \end{aligned} \tag{2.26}$$

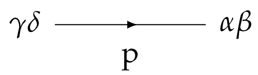
Finally with $p_1=p$ and $p_2=-p$ we have that

$$\Gamma^2_{0\;\alpha\beta\gamma\delta}(p,-p) = [p^2-m^2]1^{\alpha\beta\gamma\delta}. \tag{2.27}$$

Then the propagator for this part of the Lagrangian is given by

$$\Delta_{\alpha\beta\gamma\delta}(p) = i[\Gamma^2_0]^{-1}_{\alpha\beta\gamma\delta} = \frac{i}{p^2-m^2}1^{\alpha\beta\gamma\delta}. \tag{2.28}$$

And the Feynman diagram for this term is represented as



Third term of the Lagrangian

For the third term of the Lagrangian we start with

$$\mathcal{L} \supset -ieA^\rho[B^{\lambda\kappa\,\dagger}(T_{\rho\sigma})_{\lambda\kappa\epsilon\theta}\partial^\sigma B^{\epsilon\theta} - (\partial^\sigma B^{\lambda\kappa\,\dagger})(T_{\sigma\rho})_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta}]. \tag{2.29}$$

Then, the 3-points function is given by

$$\begin{aligned} &i\Gamma^3_{0\;\mu,\,\alpha\beta\gamma\delta}(p_1,p_2,p_3)(2\pi)^4\delta^4(p_1+p_2+p_3) = i(2\pi)^{12}\frac{\delta^3}{\delta A^\mu(p_1)\delta B^{\alpha\beta\dagger}(p_2)\delta B^{\gamma\delta}(p_3)} \\ &\times(-ie)\int d^4x\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}A^\rho(q_1)[B^{\lambda\kappa\dagger}(q_2)(T_{\rho\sigma})_{\lambda\kappa\epsilon\theta}(\partial^\sigma B^{\epsilon\theta}(q_3)) \\ &-(\partial^\sigma B^{\lambda\kappa\,\dagger}(q_2))(T_{\sigma\rho})_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta}(q_3)]e^{-iq_1\cdot x}e^{-iq_2\cdot x}e^{-iq_3\cdot x}. \end{aligned} \tag{2.30}$$

Now using again eq.(2.14) the previous expression becomes

$$\begin{aligned} &i\Gamma^3_{0\;\mu,\,\alpha\beta\gamma\delta}(p_1,p_2,p_3)(2\pi)^4\delta^4(p_1+p_2+p_3) = i\frac{\delta^3}{\delta A^\mu(p_1)\delta B^{\alpha\beta\dagger}(p_2)\delta B^{\gamma\delta}(p_3)} \\ &\times(-ie)\int d^4x d^4q_1 d^4q_2 d^4q_3 e^{-i(q_1+q_2+q_3)\cdot x}A^\rho(q_1)[B^{\lambda\kappa\dagger}(q_2)(T_{\rho\sigma})_{\lambda\kappa\epsilon\theta}(-iq_3^\sigma)B^{\epsilon\theta}(q_3) \\ &-(-iq_2^\sigma)B^{\lambda\kappa\dagger}(q_2)(T_{\sigma\rho})_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta}(q_3)]. \end{aligned} \tag{2.31}$$

Like each field is independent of the others we can compute every term of the functional derivative separately:

$$\frac{\delta A^{\rho}(q_1)}{\delta A^{\mu}(p_1)} = \delta^{\rho}_{\mu}\delta^4(q_1 - p_1), \tag{2.32}$$

$$\frac{\delta B^{\lambda\kappa\dagger}(q_2)}{\delta B^{\alpha\beta\dagger}(p_2)} = 1^{\lambda\kappa}_{\alpha\beta}\delta^4(q_2 - p_2), \tag{2.33}$$

$$\frac{\delta B^{\epsilon\theta}(q_3)}{\delta B^{\gamma\delta}(p_3)} = 1^{\epsilon\theta}_{\gamma\delta}\delta^4(q_3 - p_3). \tag{2.34}$$

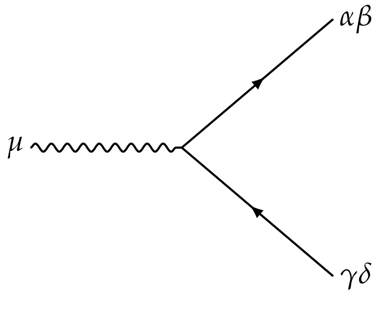
With these results we can write

$$\begin{aligned}
& i\Gamma^3_{0\ \mu,\ \alpha\beta\gamma\delta}(p_1,p_2,p_3)(2\pi)^4\delta^4(p_1+p_2+p_3) = -ie\int d^4x d^4q_1 d^4q_2 d^4q_3 \\
& \times e^{-i(q_1+q_2+q_3)\cdot x}\delta^{\rho}_{\mu}\delta^4(q_1-p_1)[1^{\lambda\kappa}_{\alpha\beta}\delta^4(q_2-p_2)(T_{\rho\sigma})_{\lambda\kappa\epsilon\theta}q_3^{\sigma}1^{\epsilon\theta}_{\gamma\delta}\delta^4(q_3-p_3) \\
& -q_2^{\sigma}1^{\lambda\kappa}_{\alpha\beta}\delta^4(q_2-p_2)(T_{\sigma\rho})_{\lambda\kappa\epsilon\theta}1^{\epsilon\theta}_{\gamma\delta}\delta^4(q_3-p_3)] \\
& = -ie\left[\int d^4x e^{-i(p_1+p_2+p_3)\cdot x}\right][(T_{\mu\nu})_{\alpha\beta\gamma\delta}p_3^{\nu} - (T_{\nu\mu})_{\alpha\beta\gamma\delta}p_2^{\nu}] \\
& = -ie(2\pi)^4\delta^4(p_1+p_2+p_3)[(T_{\mu\nu})_{\alpha\beta\gamma\delta}p_3^{\nu} - (T_{\nu\mu})_{\alpha\beta\gamma\delta}p_2^{\nu}].
\end{aligned} \tag{2.35}$$

Finally we have

$$i\Gamma^3_{0\ \mu,\ \alpha\beta\gamma\delta} = -ie[T_{\mu\nu}p_3^{\nu} - T_{\nu\mu}p_2^{\nu}]_{\alpha\beta\gamma\delta} = ie[T_{\nu\mu}p_2^{\nu} - T_{\mu\nu}p_3^{\nu}]_{\alpha\beta\gamma\delta}. \tag{2.36}$$

The Feynman diagram corresponding to the vertex given by the above expression is



Fourth term of the Lagrangian

For the fourth term of the Lagrangian we begin as the previous cases

$$\mathcal{L} \supset e^2 A^{\rho}A_{\rho}B^{\dagger\,\lambda\kappa}B_{\lambda\kappa}. \tag{2.37}$$

The 4-points function is given by

$$i\Gamma^{4\ \nu\ \ \gamma\delta}_{0\,\mu,\ \alpha\beta}(p_1,p_2,p_3,p_4)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4) = i(2\pi)^{16}\frac{\delta^4\Gamma_0}{\delta A^{\mu}(p_1)\delta A_{\nu}(p_2)\delta B^{\alpha\beta\dagger}(p_3)\delta B_{\gamma\delta}(p_4)}$$

$$= i(2\pi)^{16}e^2 \int \frac{d^4x}{(2\pi)^{16}} \left[\prod_i^4 d^4q_i\right] e^{-i(q_1+q_2+q_3+q_4)\cdot x} \frac{\delta^4[A^\rho(q_1)A_\rho(q_2)B^{\lambda\kappa\dagger}(q_3)B_{\lambda\kappa}(q_4)]}{\delta A^\mu(p_1)\delta A_\nu(p_2)\delta B^{\alpha\beta\dagger}(p_3)\delta B_{\gamma\delta}(p_4)}. \quad (2.38)$$

Now calculating the functional derivative

$$\frac{\delta^4[A^\rho(q_1)A_\rho(q_2)B^{\lambda\kappa\dagger}(q_3)B_{\lambda\kappa}(q_4)]}{\delta A^\mu(p_1)\delta A_\nu(p_2)\delta B^{\alpha\beta\dagger}(p_3)\delta B_{\gamma\delta}(p_4)} = \frac{\delta^2[A^\rho(q_1)A_\rho(q_2)]}{\delta A^\mu(p_1)\delta A_\nu(p_2)} \frac{\delta B^{\lambda\kappa\dagger}(q_3)}{\delta B^{\alpha\beta\dagger}(p_3)} \frac{\delta B_{\lambda\kappa}(q_4)}{\delta B_{\gamma\delta}(p_4)}. \quad (2.39)$$

For convenience, each part is computed separately

$$\frac{\delta B^{\lambda\kappa\dagger}(q_3)}{\delta B^{\alpha\beta\dagger}(p_3)} = 1_{\alpha\beta}^{\ \ \lambda\kappa}\delta^4(q_3-p_3), \quad (2.40)$$

$$\frac{\delta B_{\lambda\kappa}(q_4)}{\delta B_{\gamma\delta}(p_4)} = 1^{\gamma\delta}_{\ \ \lambda\kappa}\delta^4(q_4-p_4), \quad (2.41)$$

$$\begin{aligned}
\frac{\delta^2[A^\rho(q_1)A_\rho(q_2)]}{\delta A^\mu(p_1)\delta A_\nu(p_2)} &= \frac{\delta}{\delta A^\mu(p_1)}\left[\frac{\delta A^\rho(q_1)}{\delta A_\nu(p_2)}A_\rho(q_2) + A^\rho(q_1)\frac{\delta A_\rho(q_2)}{\delta A_\nu(p_2)}\right] \quad (2.42)\\
&= \frac{\delta}{\delta A^\mu(p_1)}[g^{\rho\epsilon}\delta^\nu_\epsilon\delta^4(q_1-p_2)A_\rho(q_2) + A^\rho(q_1)\delta^\nu_\rho\delta^4(q_2-p_2)]\\
&= g_{\mu\rho}g^{\rho\nu}\delta^4(q_1-p_2)\delta^4(q_2-p_1) + \delta^\nu_\mu\delta^4(q_1-p_1)\delta^4(q_2-p_2).
\end{aligned}$$

Then we have that

$$\begin{aligned}
&\frac{\delta^4[A^\rho(q_1)A_\rho(q_2)B^{\lambda\kappa\dagger}(q_3)B_{\lambda\kappa}(q_4)]}{\delta A^\mu(p_1)\delta A_\nu(p_2)\delta B^{\alpha\beta\dagger}(p_3)\delta B_{\gamma\delta}(p_4)} = \delta^\nu_\mu[\delta^4(q_1-p_2)\delta^4(q_2-p_1) + \delta^4(q_1-p_1)\delta^4(q_2-p_2)]\\
&\times 1_{\alpha\beta}^{\ \ \lambda\kappa}1^{\gamma\delta}_{\ \ \lambda\kappa}\delta^4(q_3-p_3)\delta^4(q_4-p_4)\\
&= 1_{\alpha\beta}^{\ \ \gamma\delta}\delta^\nu_\mu\delta^4(q_3-p_3)\delta^4(q_4-p_4)[\delta^4(q_1-p_2)\delta^4(q_2-p_1) + \delta^4(q_1-p_1)\delta^4(q_2-p_2)]. \quad (2.43)
\end{aligned}$$

With this result and taking into account the symmetry of the terms when the integral is computed, we can write the following

$$\begin{aligned}
&i\Gamma^{4\ \ \nu\ \gamma\delta}_{0\,\mu,\alpha\beta}(p_1,p_2,p_3,p_4)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4) = 2ie^2 1_{\alpha\beta}^{\ \ \gamma\delta}\delta^\nu_\mu \int d^4x \left[\prod_i^4 d^4q_i\right] e^{-i(q_1+q_2+q_3+q_4)\cdot x}\\
&\times\delta^4(q_1-p_1)\delta^4(q_2-p_2)\delta^4(q_3-p_3)\delta^4(q_4-p_4)\\
&= 2ie^2 1_{\alpha\beta}^{\ \ \gamma\delta}\delta^\nu_\mu \int d^4x e^{-i(p_1+p_2+p_3+p_4)\cdot x}. \quad (2.44)
\end{aligned}$$

Then it transforms into the expression

$$i\Gamma^{4\ \ \nu\ \gamma\delta}_{0\,\mu,\alpha\beta}(p_1,p_2,p_3,p_4)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4) = 2ie^2 1_{\alpha\beta}^{\gamma\delta}\delta^\nu_\mu(2\pi)^4\delta(p_1+p_2+p_3+p_4). \quad (2.45)$$

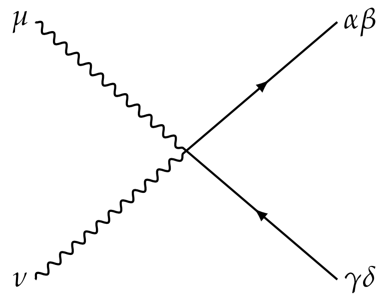
And finally we have that

$$i\Gamma_{0\,\mu,\alpha\beta}^{\ \ \ \nu\ \ \ \gamma\delta} = 2ie^2 1_{\alpha\beta}^{\ \ \gamma\delta}\delta^\nu_\mu, \quad (2.46)$$

or which is the same

$$i\Gamma_{0\,\mu\nu,\alpha\beta\gamma\delta} = 2ie^2 1_{\alpha\beta\gamma\delta}g_{\mu\nu}. \quad (2.47)$$

Then, the Feynman diagram corresponding to this term is



Self-interaction terms

Let's obtain now the Feynman rules for the self-interaction terms. Like the 4 terms have the same algebraic structure we can write them in a general form as follows

$$\mathcal{L} \supset \frac{\lambda}{2}(B^{\lambda\kappa\,\dagger}O_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta})(B^{\eta\pi\,\dagger}O_{\eta\pi\varepsilon\tau}B^{\varepsilon\tau}), \tag{2.48}$$

where the operator $O_{\lambda\kappa\epsilon\theta}$ stands for $1_{\alpha\beta\gamma\delta}$, $\chi_{\alpha\beta\gamma\delta}$, $(M^{\mu\nu})_{\alpha\beta\gamma\delta}$ and $(S^{\mu\nu})_{\alpha\beta\gamma\delta}$. Here, the 4-points function is given by

$$\begin{aligned} &i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma}(p_1,p_2,p_3,p_4)(2\pi)^4\delta^4(p_1+p_2+p_3+p_4) = i(2\pi)^{16} \\ &\times\frac{\delta^4\Gamma_0}{\delta B^{\alpha\beta\,\dagger}(p_1)\delta B^{\gamma\delta}(p_2)\delta B^{\mu\nu\,\dagger}(p_3)\delta B_{\rho\sigma}(p_4)} \\ &= i(2\pi)^{16}\frac{\lambda}{2}\int\frac{d^4x}{(2\pi)^{16}}\left[\prod_i^4 d^4q_i\right]e^{-i(q_1+q_2+q_3+q_4)\cdot x} \\ &\times\frac{\delta^4[(B^{\lambda\kappa\,\dagger}(q_1)O_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta}(q_2))(B^{\eta\pi\,\dagger}(q_3)O_{\eta\pi\varepsilon\tau}B^{\varepsilon\tau}(q_4))]}{\delta B^{\alpha\beta\,\dagger}(p_1)\delta B^{\gamma\delta}(p_2)\delta B^{\mu\nu\,\dagger}(p_3)\delta B_{\rho\sigma}(p_4)}. \end{aligned} \tag{2.49}$$

Now, determining the functional derivatives

$$\begin{aligned} &\frac{\delta^4[(B^{\lambda\kappa\,\dagger}(q_1)O_{\lambda\kappa\epsilon\theta}B^{\epsilon\theta}(q_2))(B^{\eta\pi\,\dagger}(q_3)O_{\eta\pi\varepsilon\tau}B^{\varepsilon\tau}(q_4))]}{\delta B^{\alpha\beta\,\dagger}(p_1)\delta B^{\gamma\delta}(p_2)\delta B^{\mu\nu\,\dagger}(p_3)\delta B_{\rho\sigma}(p_4)} = \\ &[O_{\mu\nu\rho\sigma}O_{\alpha\beta\gamma\delta}\delta^4(q_1-p_3)\delta^4(q_2-p_4)\delta^4(q_3-p_1)\delta^4(q_4-p_2) \\ &+O_{\alpha\beta\rho\sigma}O_{\mu\nu\gamma\delta}\delta^4(q_1-p_1)\delta^4(q_2-p_4)\delta^4(q_4-p_2)\delta^4(q_3-p_3) \\ &+O_{\mu\nu\gamma\delta}O_{\alpha\beta\rho\sigma}\delta^4(q_2-p_2)\delta^4(q_1-p_3)\delta^4(q_3-p_1))\delta^4(q_4-p_4) \\ &+O_{\alpha\beta\gamma\delta}O_{\mu\nu\rho\sigma}\delta^4(q_1-p_1)\delta^4(q_2-p_2)\delta^4(q_3-p_3)\delta^4(q_4-p_4)], \end{aligned} \tag{2.50}$$

computing the integral over q_i with the symmetry of the exponential and then integrating over x we finally obtain

$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma} = i\lambda(O_{\alpha\beta\gamma\delta}O_{\mu\nu\rho\sigma} + O_{\alpha\beta\rho\sigma}O_{\mu\nu\gamma\delta}). \tag{2.51}$$

Then we can write a Feynman rule for each term in the following way

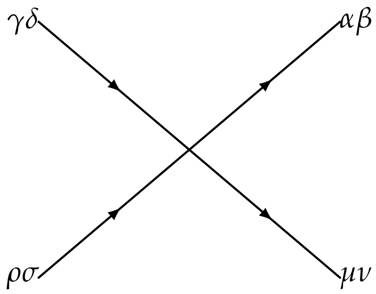
$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (1)} = i\lambda_1(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}), \tag{2.52}$$

$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (2)} = i\lambda_2(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}), \tag{2.53}$$

$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (3)} = i\lambda_3(M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}), \tag{2.54}$$

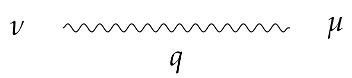
$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (4)} = i\lambda_4(S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}), \tag{2.55}$$

and the Feynman diagram for all of them has the form



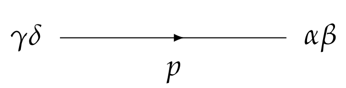
The Feynman rules for this model can be summarized as follows

(I)



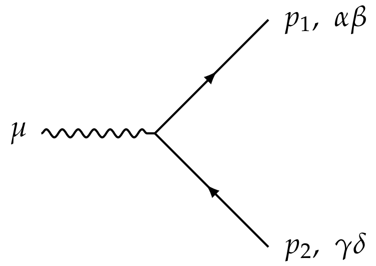
$$= -\frac{i}{q^2}[g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2}]$$

(II)



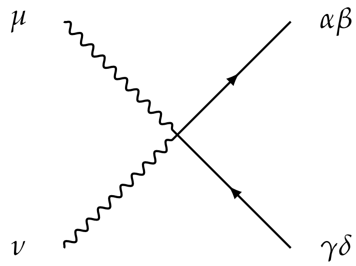
$$= \frac{i1_{\alpha\beta\gamma\delta}}{p^2 - m^2 + i\varepsilon}$$

(III)



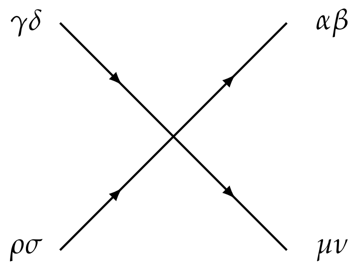
$$= -ie[T_{\mu\nu}p_2^\nu - T_{\nu\mu}p_1^\nu]_{\alpha\beta\gamma\delta}$$

(IV)



$$= 2ie^2 g_{\mu\nu} 1_{\alpha\beta\gamma\delta}$$

(V)



$$\begin{aligned} = i\{&\lambda_1(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}) \\ &+\lambda_2(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}) \\ &+\lambda_3[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}] \\ &+\lambda_4[S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\} \end{aligned}$$

Here we have changed the subscript of the momenta. The momentum p_3 is substituted by p_2 and p_2 by p_1.

2.2 Renormalization

In this section, we analyze the renormalization properties of the model at one-loop level, studying the UV divergent parts of all the potentially divergent vertex functions. In this work, we use dimensional regularization with $d = 4 - 2\epsilon$ and the naive prescription for the chirality operator χ

$$[\chi, M^{\mu\nu}] = 0, \qquad \{\chi, S^{\mu\nu}\} = 0. \tag{2.56}$$

This approach does not lead to inconsistencies as χ appears in pairs for all the processes involved. The subtraction scheme used in the study is the minimal subtraction (MS) one.

2.2.1 Counterterms

Taking eq.(2.5) as the bare Lagrangian, with all bare quantities denoted by a 0 subscript, its parameters are the tensor mass m_0, the tensor charge e_0 and the gyromagnetic factor g_0. Then it can be written as

$$\begin{aligned}
\mathcal{L}_0 \;=\;& -\frac{1}{4}F_0^{\mu\nu}F_{0\,\mu\nu} + \partial^\mu B_0^{\alpha\beta\dagger}\partial_\mu B_{0\,\alpha\beta} - m_0^2(B_0^{\alpha\beta})^\dagger B_{0\,\alpha\beta} \\
& -ie_0 A_0^\mu[B_0^{\alpha\beta\dagger}(T_{0\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_0^{\gamma\delta} - (\partial^\nu B_0^{\alpha\beta\dagger})(T_{0\,\nu\mu})_{\alpha\beta\gamma\delta}B_0^{\gamma\delta}] + e_0^2 A_0^\mu A_{0\,\mu}B_0^{\alpha\beta\dagger}B_{0\,\alpha\beta} \\
& +\frac{\lambda_{01}}{2}(B_0^{\alpha\beta\,\dagger}1_{\alpha\beta\gamma\delta}B_0^{\gamma\delta})(B_0^{\mu\nu\,\dagger}1_{\mu\nu\rho\sigma}B_0^{\rho\sigma}) + \frac{\lambda_{02}}{2}(B_0^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta}B_0^{\gamma\delta})(B_0^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma}B_0^{\rho\sigma}) \\
& +\frac{\lambda_{03}}{2}(B_0^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B_0^{\gamma_1\delta_1})(B_0^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B_0^{\gamma_2\delta_2}) \\
& +\frac{\lambda_{04}}{2}(B_0^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B_0^{\gamma_1\delta_1})(B_0^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B_0^{\gamma_2\delta_2}).
\end{aligned} \tag{2.57}$$

The renormalized fields are related to the bare ones through

$$A_r^\mu = Z_1^{-\frac{1}{2}}A_0^\mu, \qquad B_r^{\alpha\beta} = Z_2^{-\frac{1}{2}}B_0^{\alpha\beta}, \tag{2.58}$$

and therefore

$$A_0^\mu = Z_1^{\frac{1}{2}}A_r^\mu, \qquad B_0^{\alpha\beta} = Z_2^{\frac{1}{2}}B_r^{\alpha\beta}. \tag{2.59}$$

Then, the first term of the Lagrangian can be written as

$$\mathcal{L}_0^{(1)} = -\frac{1}{4}Z_1 F_r^{\mu\nu}F_{r\,\mu\nu}. \tag{2.60}$$

And with some modifications it can be obtained

$$\mathcal{L}_0^{(1)} = -\frac{1}{4}F_r^{\mu\nu}F_{r\,\mu\nu} - \frac{1}{4}(Z_1 - 1)F_r^{\mu\nu}F_{r\,\mu\nu}, \tag{2.61}$$

defining $\delta_1 \equiv Z_1 - 1$, the previous expression becomes

$$\mathcal{L}_0^{(1)} = -\frac{1}{4}F_r^{\mu\nu}F_{r\,\mu\nu} - \frac{1}{4}\delta_1 F_r^{\mu\nu}F_{r\,\mu\nu}. \tag{2.62}$$

The second term of the Lagrangian is equivalent to

$$\mathcal{L}_0^{(2)} = Z_2\partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2\frac{m_0^2}{m_r^2}Z_2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}. \tag{2.63}$$

With some modifications we get

$$\begin{aligned}
\mathcal{L}_0^{(2)} \;=\;& \partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
& +(Z_2 - 1)\partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2(Z_m - 1)(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta},
\end{aligned} \tag{2.64}$$

and finally this expression becomes

$$\begin{aligned}
\mathcal{L}_0^{(2)} \;=\;& \partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
& +\delta_2[\partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}] - \delta_m m_r^2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta},
\end{aligned} \tag{2.65}$$

where $\delta_2 \equiv Z_2 - 1$, $\delta_m \equiv Z_m - Z_2$ and $Z_m \equiv \frac{{m_0}^2}{{m_r}^2}Z_2$.

The third term can be expressed as

$$\mathcal{L}_0^{(3)} = -ie_0\frac{e_r}{e_r}Z_1^{\frac{1}{2}}Z_2A_r^{\mu}[B_r^{\alpha\beta\dagger}(T_{0\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(T_{0\,\nu\mu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}], \tag{2.66}$$

now taking $Z_e \equiv \frac{e_0}{e_r}Z_1^{\frac{1}{2}}Z_2$ this formula becomes

$$\mathcal{L}_0^{(3)} = -ie_rZ_eA_r^{\mu}[B_r^{\alpha\beta\dagger}(T_{0\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(T_{0\,\nu\mu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}]. \tag{2.67}$$

We have defined that

$$(T_{0\,\mu\nu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig_0(M_{\mu\nu})_{\alpha\beta\gamma\delta}, \tag{2.68}$$

and

$$(T_{0\,\nu\mu})_{\alpha\beta\gamma\delta} = g_{\nu\mu}1_{\alpha\beta\gamma\delta} - ig_0(M_{\nu\mu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} + ig_0(M_{\mu\nu})_{\alpha\beta\gamma\delta}. \tag{2.69}$$

Therefore it can be written

$$(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig_r\frac{g_0}{g_r}(M_{\mu\nu})_{\alpha\beta\gamma\delta}, \;\; (T_{r\,\nu\mu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} + ig_r\frac{g_0}{g_r}(M_{\mu\nu})_{\alpha\beta\gamma\delta}. \tag{2.70}$$

With this we can write eq.(2.67) as

$$\begin{aligned}\mathcal{L}_0^{(3)} &= -ie_rZ_eA_r^{\mu}[B_r^{\alpha\beta\dagger}g_{\mu\nu}1_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})g_{\mu\nu}1_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_rZ_e\frac{g_0}{g_r}A_r^{\mu}[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}],\end{aligned} \tag{2.71}$$

defining $Z_{eg} \equiv \frac{g_0}{g_r}Z_e$

$$\begin{aligned}\mathcal{L}_0^{(3)} &= -ie_rZ_eA_r^{\mu}[B_r^{\alpha\beta\dagger}g_{\mu\nu}1_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})g_{\mu\nu}1_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_rZ_{eg}A_r^{\mu}[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}].\end{aligned} \tag{2.72}$$

Now making a small modification

$$\begin{aligned}\mathcal{L}_0^{(3)} &= -ie_rA_r^{\mu}[B_r^{\alpha\beta\dagger}g_{\mu\nu}1_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})g_{\mu\nu}1_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_r(Z_e-1)A_r^{\mu}[B_r^{\alpha\beta\dagger}g_{\mu\nu}1_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})g_{\mu\nu}1_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_rA_r^{\mu}[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_r(Z_{eg}-1)A_r^{\mu}[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}].\end{aligned} \tag{2.73}$$

Defining also $\delta_e \equiv Z_e - 1$, the above expression becomes

$$\begin{aligned}\mathcal{L}_0^{(3)} &= -ie_rA_r^{\mu}[B_r^{\alpha\beta\dagger}(g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig_r(M_{\mu\nu})_{\alpha\beta\gamma\delta})\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(g_{\mu\nu}1_{\alpha\beta\gamma\delta} + (ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta})B_r^{\gamma\delta}] \\ &\quad -ie_r\delta_eA_r^{\mu}[B_r^{\alpha\beta\dagger}g_{\mu\nu}1_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})g_{\mu\nu}1_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_r(Z_{eg}-1)A_r^{\mu}[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^{\nu}B_r^{\gamma\delta} - (\partial^{\nu}B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}],\end{aligned} \tag{2.74}$$

then, with one final modification the expression becomes

$$\begin{aligned}\mathcal{L}_0^{(3)} &= -ie_r A_r^\mu [B_r^{\alpha\beta\dagger}(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta})B_r^{\gamma\delta}] \\ &\quad -ie_r\delta_e A_r^\mu [B_r^{\alpha\beta\dagger}T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})T_{r\,\nu\mu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\ &\quad -ie_r\delta_{eg} A_r^\mu [B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}],\end{aligned} \tag{2.75}$$

where have been defined $\delta_{eg} \equiv Z_{eg} - Z_e$ and $Z_{eg} \equiv \frac{g_0}{g_r} Z_e$.

The fourth term of the Lagrangian can be rewritten as

$$\mathcal{L}_0^{(4)} = e_0^2 \frac{e_r^2}{e_r^2} Z_1 Z_2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} = e_r^2 \frac{e_0^2}{e_r^2} Z_1 Z_2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta}, \tag{2.76}$$

now defining $Z_{e2} \equiv \frac{e_0^2}{e_r^2} Z_1 Z_2$ and with some modifications the above expression becomes

$$\begin{aligned}\mathcal{L}_0^{(4)} &= e_r^2(1 + Z_{e2} - 1) A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} \\ &= e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} + e_r^2 (Z_{e2} - 1) A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta},\end{aligned} \tag{2.77}$$

and defining $\delta_{e2} \equiv Z_{e2} - 1$ we finally have

$$\mathcal{L}_0^{(4)} = e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} + \delta_{e2} e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta}. \tag{2.78}$$

Now using eq.(2.59) we can write the self-interacting terms of the Lagrangian as follows

$$\mathcal{L}_{0\;si}^{(1)} = \frac{\lambda_{01}}{2} Z_2^2 (B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}), \tag{2.79}$$

and with some modifications this term becomes

$$\mathcal{L}_{0\;si}^{(1)} = \frac{\lambda_{r1}}{2} (B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) + \frac{\lambda_{r1}}{2} \delta_{\lambda 1} (B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}). \tag{2.80}$$

Similarly for the other three terms we have that

$$\mathcal{L}_{0\;si}^{(2)} = \frac{\lambda_{r2}}{2} (B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) + \frac{\lambda_{r2}}{2} \delta_{\lambda 2} (B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}), \tag{2.81}$$

$$\begin{aligned}\mathcal{L}_{0\;si}^{(3)} &= \frac{\lambda_{r3}}{2} (B_r^{\alpha_1\beta_1\,\dagger} (M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\ &\quad + \frac{\lambda_{r3}}{2} \delta_{\lambda 3} (B_r^{\alpha_1\beta_1\,\dagger} (M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),\end{aligned} \tag{2.82}$$

and

$$\begin{aligned}\mathcal{L}_{0\;si}^{(4)} &= \frac{\lambda_{r4}}{2} (B_r^{\alpha_1\beta_1\,\dagger} (S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\ &\quad + \frac{\lambda_{r4}}{2} \delta_{\lambda 4} (B_r^{\alpha_1\beta_1\,\dagger} (S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),\end{aligned} \tag{2.83}$$

where $\delta_{\lambda j} \equiv Z_{\lambda j} - 1$ and $Z_{\lambda j} \equiv \frac{\lambda_{0j}}{\lambda_{rj}} {Z_2}^2$.

Then, with all these new terms it is convenient to split the Lagrangian as the sum of two terms

$$\mathcal{L}_0 = \mathcal{L}_r + \mathcal{L}_{ct}, \tag{2.84}$$

where the first piece is the renormalized Lagrangian, and has the same structure as eq.(2.5)

$$\begin{aligned}
\mathcal{L}_r \;=\; & -\frac{1}{4} F_r^{\mu\nu} F_{r\,\mu\nu} - \frac{1}{2\xi_r}(\partial_\mu A_r^\mu)^2 + \partial^\mu B_r^{\alpha\beta\dagger} \partial_\mu B_{r\,\alpha\beta} - m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
& - i e_r A_r^\mu [B_r^{\alpha\beta\dagger} (T_{r\,\mu\nu})_{\alpha\beta\gamma\delta} \partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta}) B_r^{\gamma\delta}] \\
& + e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} + \frac{\lambda_{r1}}{2} (B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
& + \frac{\lambda_{r2}}{2} (B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
& + \frac{\lambda_{r3}}{2} (B_r^{\alpha_1\beta_1\,\dagger} (M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\
& + \frac{\lambda_{r4}}{2} (B_r^{\alpha_1\beta_1\,\dagger} (S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),
\end{aligned} \tag{2.85}$$

and the second one contains the relevant counterterms

$$\begin{aligned}
\mathcal{L}_{ct} \;=\; & -\frac{1}{4}\delta_1 F_r^{\mu\nu} F_{r\,\mu\nu} + \delta_2 [\partial^\mu B_r^{\alpha\beta\dagger} \partial_\mu B_{r\,\alpha\beta} - m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}] - \delta_m m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
& - i e_r \delta_e A_r^\mu [B_r^{\alpha\beta\dagger} (T_{r\,\mu\nu})_{\alpha\beta\gamma\delta} \partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta} B_r^{\gamma\delta}] \\
& - i e_r \delta_{eg} A_r^\mu [B_r^{\alpha\beta\dagger} (-i g_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta} \partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(i g_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta} B_r^{\gamma\delta}] \\
& + \delta_{e2} e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} + \frac{\lambda_{r1}}{2} \delta_{\lambda 1} (B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
& + \frac{\lambda_{r2}}{2} \delta_{\lambda 2} (B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
& + \frac{\lambda_{r3}}{2} \delta_{\lambda 3} (B_r^{\alpha_1\beta_1\,\dagger} (M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\
& + \frac{\lambda_{r4}}{2} \delta_{\lambda 4} (B_r^{\alpha_1\beta_1\,\dagger} (S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1})(B_r^{\alpha_2\beta_2\,\dagger} (S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),
\end{aligned} \tag{2.86}$$

where we have used the following definitions

$$\begin{array}{llll}
\delta_1 \equiv Z_1 - 1, & \delta_2 \equiv Z_2 - 1, & \delta_m \equiv Z_m - Z_2, & \delta_e \equiv Z_e - 1, \\
\delta_{eg} \equiv Z_{eg} - Z_e, & \delta_{e2} \equiv Z_{e2} - 1, & \delta_{\lambda j} \equiv Z_{\lambda j} - 1, & \xi_r \equiv Z_1^{-1} \xi_0,
\end{array} \tag{2.87}$$

and

$$Z_m \equiv \frac{m_0^2}{m_r^2} Z_2, \quad Z_e \equiv \frac{e_0}{e_r} Z_1^{\frac{1}{2}} Z_2, \quad Z_{eg} \equiv \frac{g_0}{g_r} Z_e, \quad Z_{e2} \equiv \frac{e_0^2}{e_r^2} Z_1 Z_2 \quad Z_{\lambda j} \equiv \frac{\lambda_{0j}}{\lambda_{rj}} {Z_2}^2. \tag{2.88}$$

2.2.2 Determining the Feynman rules for the counterterms

The Feynman rules for the counterterms are very similar to the rules for the bare Lagrangian.

For the first term of the Lagrangian (2.86)

$$\mathcal{L}_{ct} \supset -\frac{1}{4}\delta_1 F_r^{\mu\nu} F_{r\,\mu\nu}, \tag{2.89}$$

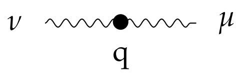
the 2-points function reads

$$\Gamma^2_{0\mu\nu}(q,-q) = -\delta_1[q^2 g_{\mu\nu} - q_\mu q_\nu]. \tag{2.90}$$

Then, the propagator is

$$\Delta_{\mu\nu}(q) = i\Gamma^2_{0\mu\nu}(q,-q) = -i\delta_1[q^2 g_{\mu\nu} - q_\mu q_\nu]. \tag{2.91}$$

And the Feynman diagram for this one is depicted by



For the second term

$$\mathcal{L}_{ct} \supset \delta_2(\partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} - m_r^2(B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}) - \delta_m m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}, \tag{2.92}$$

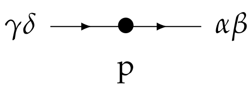
the 2-points function reads

$$\Gamma^2_{0\;\alpha\beta\gamma\delta}(p,-p) = [\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}, \tag{2.93}$$

and the propagator is

$$\Delta(p) = i\Gamma^2_{0\;\alpha\beta\gamma\delta}(p,-p) = i[\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}. \tag{2.94}$$

With the Feynman diagram given by



For the third term of the Lagrangian

$$\mathcal{L}_{ct} \supset -ie_r\delta_e A_r^\mu[B_r^{\alpha\beta\dagger}(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta} B_r^{\gamma\delta}], \tag{2.95}$$

the 3-points function reads

$$i\Gamma^3_{0\;\mu,\,\alpha\beta\gamma\delta} = -ie_r\delta_e[(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}p_3^\nu - p_2^\nu(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta}]. \tag{2.96}$$

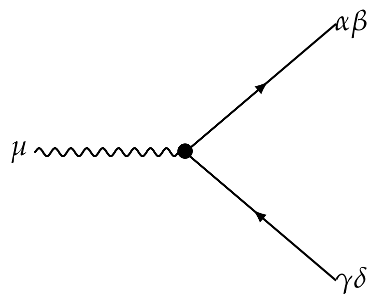
For the fourth one

$$\mathcal{L}_{ct} \supset -ie_r\delta_{eg}A_r^\mu[B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} - (\partial^\nu B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}], \tag{2.97}$$

the 3-points function is given by

$$i\Gamma^3_{0\ \mu,\ \alpha\beta\gamma\delta} = -ie_r\delta_{eg}[(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}p_3^\nu - p_2^\nu(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}]. \quad (2.98)$$

And the Feynman diagram for these two last terms is then represented by



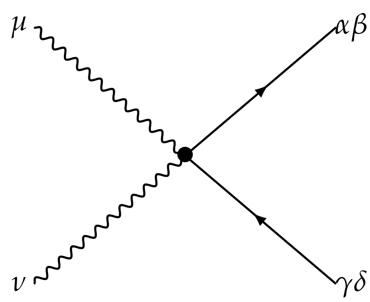
For the fifth term

$$\mathcal{L}_{ct} \supset \delta_{e2}e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta}, \quad (2.99)$$

the 4-points function reads

$$i\Gamma^4_{0\,\mu\nu,\alpha\beta\gamma\delta} = 2ie_r^2\delta_{e2}1_{\alpha\beta\gamma\delta}g_{\mu\nu}. \quad (2.100)$$

And the Feynman diagram for this term is



The Feynman rules for the self-interaction terms are exactly the same as for these terms in the bare Lagrangian except for the multiplication of the constants $\delta_{\lambda j}$. Then they are given by

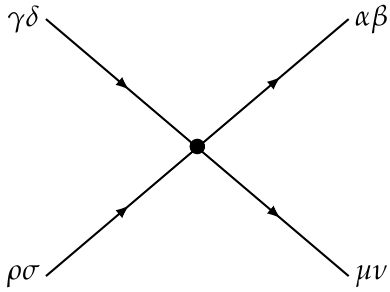
$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (1)} = i\lambda_{r1}\delta_{\lambda 1}(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}), \quad (2.101)$$

$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (2)} = i\lambda_{r2}\delta_{\lambda 2}(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}), \quad (2.102)$$

$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (3)} = i\lambda_{r3}\delta_{\lambda 3}(M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}), \quad (2.103)$$

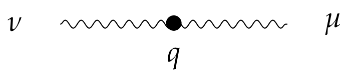
$$i\Gamma^4_{0\ \alpha\beta\gamma\delta,\ \mu\nu\rho\sigma\ (4)} = i\lambda_{r4}\delta_{\lambda 4}(S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}), \quad (2.104)$$

and the Feynman diagram for all of them is



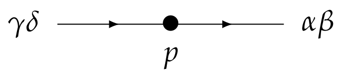
The Feynman rules for the counterterms can be summarized as follows

(VI)



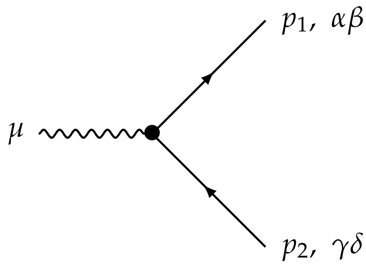
$$= -i\delta_1[q^2 g_{\mu\nu} - q_\mu q_\nu]$$

(VII)



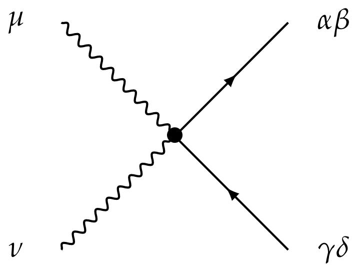
$$= i[\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}$$

(VIII)



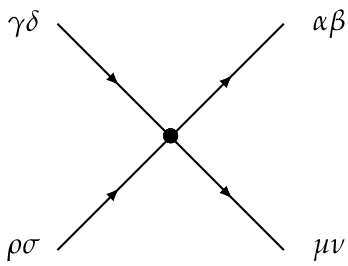
$$= -ie\delta_e[T_{\mu\nu}p_2^\nu - T_{\nu\mu}p_1^\nu]_{\alpha\beta\gamma\delta}$$
$$-eg\delta_{eg}(p_1^\nu + p_2^\nu)(M_{\mu\nu})_{\alpha\beta\gamma\delta}$$

(IX)



$$= 2ie^2\delta_{e2}1_{\alpha\beta\gamma\delta}g_{\mu\nu}$$

(X)



$$\begin{aligned} = i\{&\lambda_1\delta_{\lambda 1}(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}) \\ &+\lambda_2\delta_{\lambda 2}(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}) \\ &+\lambda_3\delta_{\lambda 3}[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}] \\ &+\lambda_4\delta_{\lambda 4}[S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\} \end{aligned}$$

Here we have also made the changes expressed for the Feynman rules of the model.

Chapter 3

Simplified model of self-interactions

In this Chapter we will carry out the renormalization of a simplified model where only the self-interaction terms are considered.

It is important to remark that all the calculations of this Chapter and the ones of Chapter 4 are going to be developed using the package FeynCalc [66, 67] of the software Wolfram Mathematica and that we are working in $d = 4 - 2\epsilon$ dimensions. Then, the renormalized parameters must be scaled according to

$$e_r \to \mu^{\epsilon} e_r, \quad g_r \to g_r, \quad \lambda_{r\,i} \to \mu^{2\epsilon}\lambda_{r\,i}, \quad m_r \to m_r, \tag{3.1}$$

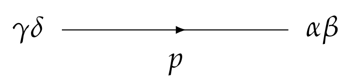
where μ is the arbitrary scale introduced by dimensional regularization. In what follows, we will omit the r subscript for the renormalized parameters.

3.1 Presentation of the terms

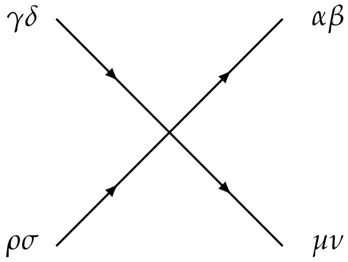
We are going to work with a part of the total Lagrangian, which is given by

$$\begin{aligned}
\mathcal{L}_s &= \partial^{\mu} B^{\alpha\beta\dagger}\partial_{\mu} B_{\alpha\beta} - m^2 (B^{\alpha\beta})^{\dagger} B_{\alpha\beta} + \frac{\lambda_1}{2}(B^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B^{\gamma\delta})(B^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B^{\rho\sigma}) \\
&+ \frac{\lambda_2}{2}(B^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta} B^{\gamma\delta})(B^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma} B^{\rho\sigma}) + \frac{\lambda_3}{2}(B^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B^{\gamma_2\delta_2}) \\
&+ \frac{\lambda_4}{2}(B^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B^{\gamma_2\delta_2}) + \delta_2[\partial^{\mu} B^{\alpha\beta\dagger}\partial_{\mu} B_{\alpha\beta} - m^2 (B^{\alpha\beta})^{\dagger} B_{\alpha\beta}] \\
&- \delta_m m^2 (B^{\alpha\beta})^{\dagger} B_{\alpha\beta} + \frac{\lambda_1}{2}\delta_{\lambda 1}(B^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B^{\gamma\delta})(B^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B^{\rho\sigma}) \\
&+ \frac{\lambda_2}{2}\delta_{\lambda 2}(B^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta} B^{\gamma\delta})(B^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma} B^{\rho\sigma}) \\
&+ \frac{\lambda_3}{2}\delta_{\lambda 3}(B^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B^{\gamma_2\delta_2}) \\
&+ \frac{\lambda_4}{2}\delta_{\lambda 4}(B^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B^{\gamma_1\delta_1})(B^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B^{\gamma_2\delta_2}),
\end{aligned} \tag{3.2}$$

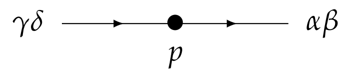
and the Feynman rules for these terms can be summarized as follows



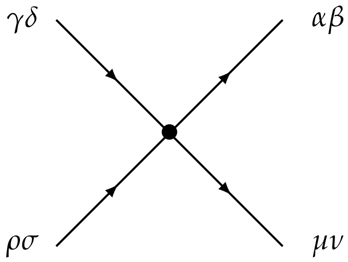
$$= \frac{i1_{\alpha\beta\gamma\delta}}{p^2 - m^2 + i\varepsilon}$$



$$\begin{aligned} = i\{&\lambda_1(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}) \\ &+\lambda_2(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}) \\ &+\lambda_3[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}] \\ &+\lambda_4[S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\} \end{aligned}$$



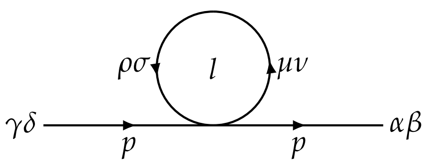
$$= i[\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}$$



$$\begin{aligned} = i\{&\lambda_1\delta_{\lambda 1}(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}) \\ &+\lambda_2\delta_{\lambda 2}(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}) \\ &+\lambda_3\delta_{\lambda 3}[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}] \\ &+\lambda_4\delta_{\lambda 4}[S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\} \end{aligned}$$

3.2 Tensor self-energy

In this model we have to calculate one correction for the tensor self-energy at one-loop order. This one is given by



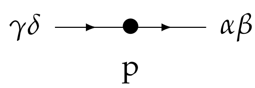
And its amplitude is given by

$$\begin{aligned} -i\Sigma_{\alpha\beta\gamma\delta}(p)_s = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\frac{1}{l^2 - m^2}\right][\lambda_1(1_{\alpha\beta\gamma\delta}1_{\mu\nu}{}^{\mu\nu} + 1_{\alpha\beta}{}^{\mu\nu}1_{\mu\nu\gamma\delta}) \\ +\lambda_2(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu}{}^{\mu\nu} + \chi_{\alpha\beta}{}^{\mu\nu}\chi_{\mu\nu\gamma\delta}) + \lambda_3(M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu}{}^{\mu\nu} + M^{\kappa\lambda}{}_{\alpha\beta}{}^{\mu\nu}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}) \\ +\lambda_4(S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu}{}^{\mu\nu} + S^{\kappa\lambda}{}_{\alpha\beta}{}^{\mu\nu}(S_{\kappa\lambda})_{\mu\nu\gamma\delta})]. \end{aligned} \tag{3.3}$$

The divergent part of this amplitude reads

$$-i\Sigma^*_{\alpha\beta\gamma\delta}(p)_s = \left[-im^2\{\frac{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4}{16\pi^2\epsilon}\}\right]1_{\alpha\beta\gamma\delta}. \tag{3.4}$$

The counterterm is



with an amplitude

$$-i\Sigma_{\alpha\beta\gamma\delta}(p)_{s\,ct} = i[\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}. \tag{3.5}$$

Then the counterterms that cancel the UV divergence are given by

$$\delta_2 = 0, \qquad \delta_m = -\frac{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4}{16\pi^2\epsilon}. \tag{3.6}$$

3.3 TTTT vertex

In this particular case we have to calculate three corrections (s, t and u channels) for the TTTT vertex. The name of these channels comes from the Mandelstam variables. These variables are numerical quantities that encode the energy, momentum, and angles of particles in scattering processes. They are used for scattering processes of two particles to two particles and are defined as

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2, \tag{3.7}$$

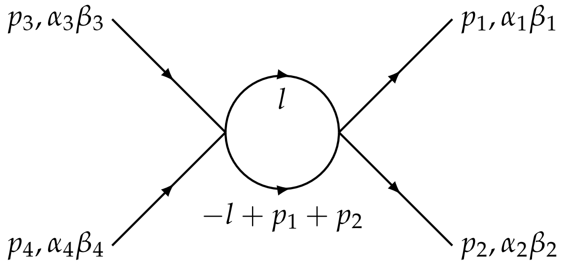
$$t = (p_1 - p_3)^2 = (p_4 - p_2)^2, \tag{3.8}$$

$$u = (p_1 - p_4)^2 = (p_3 - p_2)^2, \tag{3.9}$$

where p_1 and p_2 are the four-momenta of the incoming particles and p_3 and p_4 are the four-momenta of the outgoing particles. The s-channel, t-channel, u-channel represent different Feynman diagrams or different possible scattering events.

For example, the s-channel corresponds to the process where the incoming particles 1 and 2 joining into an intermediate particle that later splits into particles 3 and 4. The t-channel represents the process in which the particle 1 emits the intermediate particle and becomes the final particle 3, while the particle 2 absorbs the intermediate particle and becomes 4. The u-channel is the t-channel but particle 1 becomes 4 and particle 2 becomes 3.

The one-loop correction for the s-channel is given by

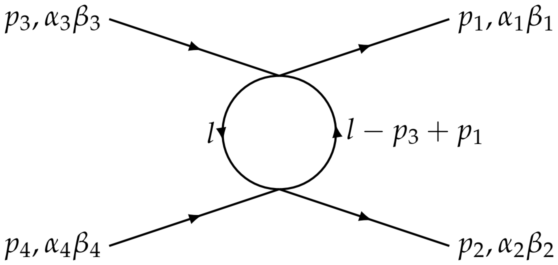


and its amplitude is

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,s\,1} = \mu^{2\varepsilon}\int \frac{d^dl}{(2\pi)^d} i[\lambda_1(1_{\alpha_1\beta_1\gamma_3\delta_3}1_{\alpha_2\beta_2\gamma_4\delta_4} + 1_{\alpha_1\beta_1\gamma_4\delta_4}1_{\alpha_2\beta_2\gamma_3\delta_3})$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\gamma_3\delta_3}\chi_{\alpha_2\beta_2\gamma_4\delta_4} + \chi_{\alpha_1\beta_1\gamma_4\delta_4}\chi_{\alpha_2\beta_2\gamma_3\delta_3}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})]$$

$$\times \left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right] i[\lambda_1(1_{\gamma_1\delta_1\alpha_3\beta_3}1_{\gamma_2\delta_2\alpha_4\beta_4}+1_{\gamma_1\delta_1\alpha_4\beta_4}1_{\gamma_2\delta_2\alpha_3\beta_3})+\lambda_2(\chi_{\gamma_1\delta_1\alpha_3\beta_3}\chi_{\gamma_2\delta_2\alpha_4\beta_4}$$
$$+\chi_{\gamma_1\delta_1\alpha_4\beta_4}\chi_{\gamma_2\delta_2\alpha_3\beta_3})+\lambda_3(M^{\kappa\lambda}_{\gamma_1\delta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_2\delta_2\alpha_4\beta_4}+M^{\kappa\lambda}_{\gamma_1\delta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\gamma_2\delta_2\alpha_3\beta_3})$$
$$+\lambda_4(S^{\kappa\lambda}_{\gamma_1\delta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_2\delta_2\alpha_4\beta_4}+S^{\kappa\lambda}_{\gamma_1\delta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\gamma_2\delta_2\alpha_3\beta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{(p_1+p_2-l)^2-m^2}\right]. \quad (3.10)$$

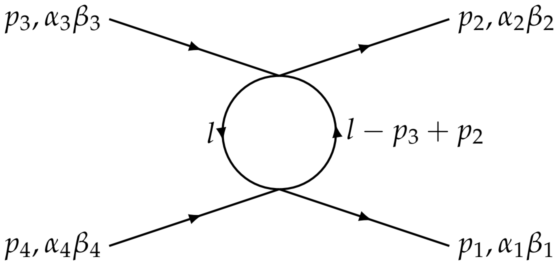
The correction for the t-channel is given by



with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,s\,2}=\mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}i[\lambda_1(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\gamma_1\delta_1\gamma_4\delta_4}+1_{\alpha_1\beta_1\gamma_4\delta_4}1_{\gamma_1\delta_1\alpha_3\beta_3})$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\gamma_1\delta_1\gamma_4\delta_4}+\chi_{\alpha_1\beta_1\gamma_4\delta_4}\chi_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(M_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}+S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(S_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})]$$
$$\times\left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right]i[\lambda_1(1_{\gamma_2\delta_2\gamma_3\delta_3}1_{\alpha_2\beta_2\alpha_4\beta_4}+1_{\gamma_2\delta_2\alpha_4\beta_4}1_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_2(\chi_{\gamma_2\delta_2\gamma_3\delta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+\chi_{\gamma_2\delta_2\alpha_4\beta_4}\chi_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}+M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})$$
$$+\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}+S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{(l-p_3+p_1)^2-m^2}\right]. \quad (3.11)$$

And the correction for the u-channel is given by



with amplitude

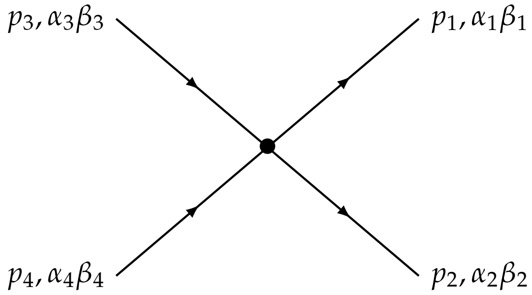
$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,s\,3}=\mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}i[\lambda_1(1_{\alpha_2\beta_2\alpha_3\beta_3}1_{\gamma_1\delta_1\gamma_4\delta_4}+1_{\alpha_2\beta_2\gamma_4\delta_4}1_{\gamma_1\delta_1\alpha_3\beta_3})$$
$$+\lambda_2(\chi_{\alpha_2\beta_2\alpha_3\beta_3}\chi_{\gamma_1\delta_1\gamma_4\delta_4}+\chi_{\alpha_2\beta_2\gamma_4\delta_4}\chi_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_3(M^{\kappa\lambda}_{\alpha_2\beta_2\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}$$
$$+M^{\kappa\lambda}_{\alpha_2\beta_2\gamma_4\delta_4}(M_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_4(S^{\kappa\lambda}_{\alpha_2\beta_2\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}+S^{\kappa\lambda}_{\alpha_2\beta_2\gamma_4\delta_4}(S_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})]$$
$$\times\left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right]i[\lambda_1(1_{\gamma_2\delta_2\gamma_3\delta_3}1_{\alpha_1\beta_1\alpha_4\beta_4}+1_{\gamma_2\delta_2\alpha_4\beta_4}1_{\alpha_1\beta_1\gamma_3\delta_3})+\lambda_2(\chi_{\gamma_2\delta_2\gamma_3\delta_3}\chi_{\alpha_1\beta_1\alpha_4\beta_4}$$

$$+\chi_{\gamma_2\delta_2\alpha_4\beta_4}\chi_{\alpha_1\beta_1\gamma_3\delta_3}) + \lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_1\beta_1\alpha_4\beta_4} + M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_1\beta_1\gamma_3\delta_3})$$
$$+\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_1\beta_1\alpha_4\beta_4} + S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_1\beta_1\gamma_3\delta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{(l-p_3+p_2)^2-m^2}\right]. \qquad (3.12)$$

The divergent part of the TTTT vertex is

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,s} = \frac{1}{16\pi^2\epsilon}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2 + 8\lambda_3 + 12\lambda_4\right) + 3({\lambda_2}^2 + 16{\lambda_3}^2)$$
$$-8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3+\lambda_4\right)\Big\}(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\alpha_2\beta_2\alpha_4\beta_4} + 1_{\alpha_1\beta_1\alpha_4\beta_4}1_{\alpha_2\beta_2\alpha_3\beta_3})$$
$$+\frac{1}{2\pi^2\epsilon}\Big\{\lambda_2(\lambda_1+\lambda_2) + 2\lambda_2\left(\lambda_3-\lambda_4\right) + 6\left(\lambda_3-\lambda_4\right)^2\Big\}(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+\chi_{\alpha_1\beta_1\alpha_4\beta_4}\chi_{\alpha_2\beta_2\alpha_3\beta_3}) - \frac{1}{2\pi^2\epsilon}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1+\lambda_2+\lambda_3\right)\Big\}(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}) + \frac{1}{2\pi^2\epsilon}\Big\{\lambda_4(\lambda_1 - 4\lambda_3 + 2\lambda_4)\Big\}(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}). \qquad (3.13)$$

The counterterm for this vertex is given by



and its amplitude is

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,s\,ct} = i[\lambda_1\delta_{\lambda 1}(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\alpha_2\beta_2\alpha_4\beta_4} + 1_{\alpha_1\beta_1\alpha_4\beta_4}1_{\alpha_2\beta_2\alpha_3\beta_3}) \qquad (3.14)$$
$$+\lambda_2\delta_{\lambda 2}(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4} + \chi_{\alpha_1\beta_1\alpha_4\beta_4}\chi_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_3\delta_{\lambda 3}(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_4\delta_{\lambda 4}(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} + S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3})].$$

The corresponding counterterms that render the total amplitude finite are given in the MS scheme by

$$\delta_{\lambda 1} = -\frac{1}{16\pi^2\lambda_1\epsilon}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2+8\lambda_3+12\lambda_4\right) + 3({\lambda_2}^2 + 16{\lambda_3}^2) \qquad (3.15)$$
$$-8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3+\lambda_4\right)\Big\},$$
$$\delta_{\lambda 2} = -\frac{1}{2\pi^2\lambda_2\epsilon}\Big\{\lambda_2(\lambda_1+\lambda_2) + 2\lambda_2\left(\lambda_3-\lambda_4\right) + 6\left(\lambda_3-\lambda_4\right)^2\Big\}, \qquad (3.16)$$
$$\delta_{\lambda 3} = \frac{1}{2\pi^2\lambda_3\epsilon}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1+\lambda_2+\lambda_3\right)\Big\}, \qquad (3.17)$$
$$\delta_{\lambda 4} = -\frac{1}{2\pi^2\epsilon}\Big\{\lambda_1 - 4\lambda_3 + 2\lambda_4\Big\}. \qquad (3.18)$$

3.4 Beta functions

In this section we are going to present the beta functions for the simplified model we have been analyzing in this chapter.

Summarizing, from the results obtained in eqs.(3.6,3.15,3.16,3.17,3.18) and the definition of the counterterms in eqs.(2.87,2.88), the relations between the bare and renormalized parameters of the theory are given by

$$\lambda_{0j} = Z_2^{-2} Z_{\lambda_j} \mu^{2\epsilon} \lambda_j, \qquad m_0^2 = Z_2^{-1} Z_m m^2, \tag{3.19}$$

the renormalization constants defined in the MS scheme are

$$\begin{aligned}
Z_{\lambda_1}^{\text{MS}} &= 1 - \frac{1}{16\pi^2 \lambda_1 \epsilon}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2 + 8\lambda_3 + 12\lambda_4\right) \\
&\quad + 3({\lambda_2}^2 + 16{\lambda_3}^2) - 8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3 + \lambda_4\right)\Big\}, &(3.20)\\
Z_{\lambda_2}^{\text{MS}} &= 1 - \frac{1}{2\pi^2 \lambda_2 \epsilon}\Big\{\lambda_2(\lambda_1 + \lambda_2) + 2\lambda_2\left(\lambda_3 - \lambda_4\right) + 6\left(\lambda_3 - \lambda_4\right)^2\Big\}, &(3.21)\\
Z_{\lambda_3}^{\text{MS}} &= 1 + \frac{1}{2\pi^2 \lambda_3 \epsilon}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1 + \lambda_2 + \lambda_3\right)\Big\}, &(3.22)\\
Z_{\lambda_4}^{\text{MS}} &= 1 - \frac{1}{2\pi^2 \epsilon}\Big\{\lambda_1 - 4\lambda_3 + 2\lambda_4\Big\}, &(3.23)\\
Z_m^{\text{MS}} &= Z_2^{\text{MS}} + \delta_m^{\text{MS}} = 1 - \frac{1}{16\pi^2 \epsilon}\Big\{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. &(3.24)
\end{aligned}$$

From eqs.(3.19-3.24) one can extract the following beta functions[1] $\beta_\eta \equiv \mu \frac{\partial \eta}{\partial \mu}$ and anomalous dimensions $\gamma_m \equiv \frac{\mu}{m}\frac{\partial m}{\partial \mu}$ in the $\epsilon \to 0$ limit:

$$\begin{aligned}
\beta_{\lambda 1} &= -\frac{1}{8\pi^2}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2 + 8\lambda_3 + 12\lambda_4\right) + 3\lambda_2^2 + 48\lambda_3^2 \\
&\quad - 8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3 + \lambda_4\right)\Big\}, &(3.25)\\
\beta_{\lambda 2} &= -\frac{1}{\pi^2}\Big\{\lambda_2^2 + \lambda_1\lambda_2 + 2\lambda_2\left(\lambda_3 - \lambda_4\right) + 6\left(\lambda_3 - \lambda_4\right)^2\Big\}, &(3.26)\\
\beta_{\lambda 3} &= \frac{1}{\pi^2}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1 + \lambda_2 + \lambda_3\right)\Big\}, &(3.27)\\
\beta_{\lambda 4} &= -\frac{1}{\pi^2}\Big\{\lambda_4\left(\lambda_1 - 4\lambda_3 + 2\lambda_4\right)\Big\}, &(3.28)\\
\gamma_m &= -\frac{1}{16\pi^2}\Big\{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. &(3.29)
\end{aligned}$$

[1] The general procedure to determine the beta functions is presented in Appendix B

Chapter 4

Quantum Electrodynamics

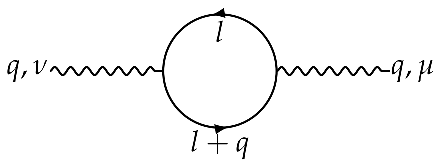
In this Chapter we study the renormalization of the Quantum Electrodynamics of the complete model using the same tools and conventions[1] of Chapter 3. Also, there will be determined the beta functions of the model and some fixed points are going to be presented.

4.1 Vacuum Polarization

In this model there are two diagrams contributing to the vacuum polarization at one-loop order. Then, we can write

$$-i\Pi^{\mu\nu}(q) = -i\Pi^{\mu\nu}(q)_1 - i\Pi^{\mu\nu}(q)_2. \tag{4.1}$$

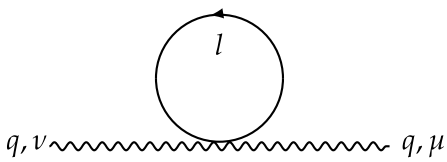
The first one is



and its amplitude is given by

$$\begin{aligned} -i\Pi^{\mu\nu}(q)_1 = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\nu}(-l-q)^\rho - T_{\nu\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1} \\ \times \left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{(l+q)^2 - m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}(l+q)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2 - m^2}\right], \end{aligned} \tag{4.2}$$

and the second one is



[1]These tools and conventions are briefly presented at the beginning of Chapter 3.

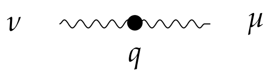
with amplitude

$$-i\Pi^{\mu\nu}(q)_2 = \mu^{2\varepsilon}\left[\int \frac{d^dl}{(2\pi)^d}\right] 2ie^2 g_{\mu\nu} 1_{\alpha\beta\gamma\delta}\left[\frac{i1^{\gamma\delta\alpha\beta}}{l^2-m^2}\right]. \tag{4.3}$$

The divergent piece, denoted by $-i\Pi^{\mu\nu}(q)^*$ is given by

$$-i\Pi^{\mu\nu}(q)^* = i\frac{e^2(2g^2-1)}{8\pi^2\epsilon}(q^2g^{\mu\nu}-q^\mu q^\nu). \tag{4.4}$$

The counterterm is represented by



and its amplitude is

$$-i\Pi^{\mu\nu}(q)_{ct} = -i\delta_1[q^2g_{\mu\nu}-q_\mu q_\nu]. \tag{4.5}$$

The divergent piece in eq.(4.4) can be removed in the MS scheme by fixing the counterterm δ_1 as

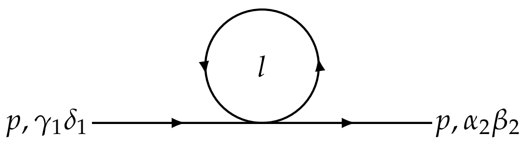
$$\delta_1 = \frac{e^2(2g^2-1)}{8\pi^2\epsilon}. \tag{4.6}$$

4.2 Tensor self-energy

In the complete model we have to calculate three corrections for this propagator. Then, we can write

$$-i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p) = -i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_1 - i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_2 - i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_3. \tag{4.7}$$

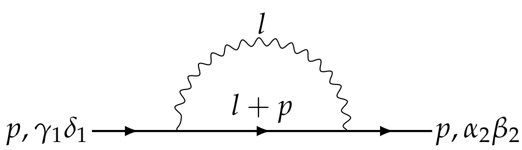
The first contribution is given by the diagram



whose amplitude is

$$\begin{aligned}-i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_1 = \mu^{2\varepsilon}\left[\int \frac{d^dl}{(2\pi)^d}\frac{1}{l^2-m^2}\right][\lambda_1(1_{\alpha_2\beta_2\gamma_1\delta_1}1_{\mu\nu}{}^{\mu\nu} + 1_{\alpha_2\beta_2}{}^{\mu\nu}1_{\mu\nu\gamma_1\delta_1})\\ +\lambda_2(\chi_{\alpha_2\beta_2\gamma_1\delta_1}\chi_{\mu\nu}{}^{\mu\nu} + \chi_{\alpha_2\beta_2}{}^{\mu\nu}\chi_{\mu\nu\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_2\beta_2\gamma_1\delta_1}(M_{\kappa\lambda})_{\mu\nu}{}^{\mu\nu}\\ +M^{\kappa\lambda}{}_{\alpha_2\beta_2}{}^{\mu\nu}(M_{\kappa\lambda})_{\mu\nu\gamma_1\delta_1}) + \lambda_4(S^{\kappa\lambda}_{\alpha_2\beta_2\gamma_1\delta_1}(S_{\kappa\lambda})_{\mu\nu}{}^{\mu\nu} + S^{\kappa\lambda}{}_{\alpha_2\beta_2}{}^{\mu\nu}(S_{\kappa\lambda})_{\mu\nu\gamma_1\delta_1})].\end{aligned} \tag{4.8}$$

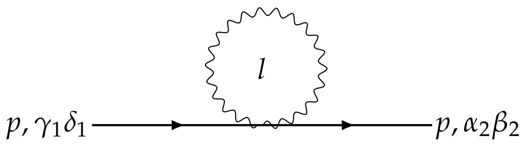
The second one is



with an amplitude

$$-i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_2 = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\mu}(-l-p)^\rho - T_{\mu\rho}p^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[-\frac{i}{l^2}\left(g^{\mu\nu}+(\xi-1)\frac{l^\mu l^\nu}{l^2}\right)\right] ie[T_{\rho\nu}(-p)^\rho - T_{\nu\rho}(l+p)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{(l+p)^2-m^2}\right], \quad (4.9)$$

and the third one



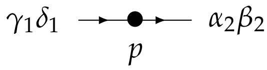
with an amplitude

$$-i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_3 = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\nu} 1_{\alpha_2\beta_2\gamma_1\delta_1}\left[-\frac{i}{l^2}\left(g^{\mu\nu}+(\xi-1)\frac{l^\mu l^\nu}{l^2}\right)\right]. \quad (4.10)$$

The divergent part of this amplitude is given by

$$-i\Sigma^*_{\alpha_2\beta_2\gamma_1\delta_1}(p) = \frac{-i}{32\pi^2\epsilon}\Big\{m^2\left(2e^2g^2+e^2\xi+7\lambda_1+\lambda_2+8\lambda_3+12\lambda_4\right)$$
$$-e^2(\xi-3)p^2\Big\}1_{\alpha_2\beta_2\gamma_1\delta_1}. \quad (4.11)$$

The counterterm is represented by



and its amplitude is

$$i\Sigma_{\alpha_2\beta_2\gamma_1\delta_1}(p)_{ct} = i[\delta_2(p^2-m^2)-\delta_m m^2]1_{\alpha_2\beta_2\gamma_1\delta_1}. \quad (4.12)$$

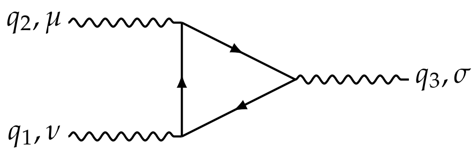
Then the values of the counterterms that cancel the UV divergence are then given by

$$\delta_2 = -\frac{e^2(\xi-3)}{16\pi^2\epsilon}, \quad (4.13)$$
$$\delta_m = -\frac{e^2\left(2g^2+3\right)+7\lambda_1+\lambda_2+8\lambda_3+12\lambda_4}{16\pi^2\epsilon}. \quad (4.14)$$

4.3 $\gamma\gamma\gamma$ vertex

For the $\gamma\gamma\gamma$ vertex we have to determine two contributions.

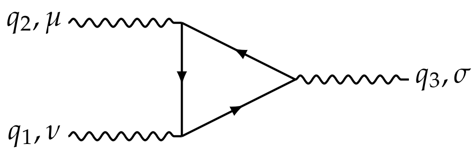
The first correction is



with an amplitude

$$-i\mathcal{M}_{\sigma\mu\nu\,1} = \mu^{2\varepsilon}\left[\int \frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}(l-q_1)^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l-q_2)^\rho - T_{\mu\rho}(l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{(l+q_2)^2-m^2}\right]$$
$$\times ie[T_{\rho\sigma}(q_1-l)^\rho - T_{\sigma\rho}(l+q_2)^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{(l-q_1)^2-m^2}\right], \tag{4.15}$$

and the second one is



with an amplitude

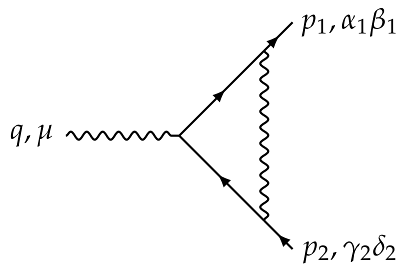
$$-i\mathcal{M}_{\sigma\mu\nu\,2} = \mu^{2\varepsilon}\left[\int \frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\nu}(l-q_1)^\rho - T_{\nu\rho}(-l)^\rho]_{\gamma_1\delta_1\alpha_1\beta_1}$$
$$\times\left[\frac{i1^{\alpha_1\beta_1\gamma_2\delta_2}}{l^2-m^2}\right] ie[T_{\rho\mu}(l)^\rho - T_{\mu\rho}(-l-q_2)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2}\left[\frac{i1^{\alpha_2\beta_2\gamma_3\delta_3}}{(l+q_2)^2-m^2}\right]$$
$$\times ie[T_{\rho\sigma}(l+q_2)^\rho - T_{\sigma\rho}(q_1-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\alpha_3\beta_3\gamma_1\delta_1}}{(l-q_1)^2-m^2}\right]. \tag{4.16}$$

The contribution to the $\gamma\gamma\gamma$ vertex from the diagrams shown above vanishes identically from the charge conjugation invariance of the theory.

$$-i\mathcal{M}_{\sigma\mu\nu} = -i\mathcal{M}_{\sigma\mu\nu\,1} - i\mathcal{M}_{\sigma\mu\nu\,2} = 0. \tag{4.17}$$

4.4 TTγ vertex

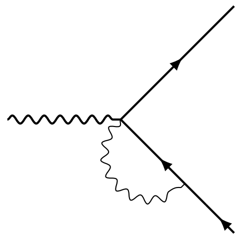
The corrections to the TTγ vertex are given by four diagrams. The first one is



with an amplitude

$$-ie\Gamma^{\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2)_1 = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\nu}(p_1)^\rho - T_{\nu\rho}(-p_1-l)^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{(-p_1-l)^2-m^2}\right] ie[T_{\rho\mu}(l+p_1)^\rho - T_{\mu\rho}(p_2-l)^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{(l-p_2)^2-m^2}\right]$$
$$\times ie[T_{\rho\sigma}(l-p_2)^\rho - T_{\sigma\rho}(p_2)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\nu}+(\xi-1)\frac{l^\sigma l^\nu}{l^2}\right)\right]. \tag{4.18}$$

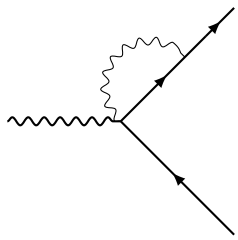
The second one is



with an amplitude

$$-ie\Gamma^{\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2)_2 = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] 2ie^2 g_{\mu\nu}1_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.19}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{(l-p_2)^2-m^2}\right]\left[-\frac{i}{l^2}\left(g^{\nu\sigma}+(\xi-1)\frac{l^\nu l^\sigma}{l^2}\right)\right] ie[T_{\rho\sigma}(l-p_2)^\rho - T_{\sigma\rho}(p_2)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}.$$

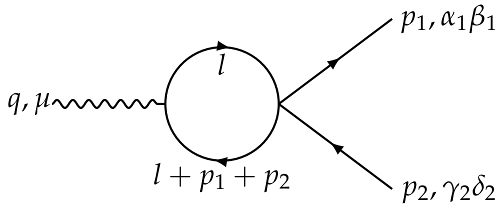
The third one is



with an amplitude

$$-ie\Gamma^{\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2)_3 = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\sigma}(p_1)^\rho - T_{\sigma\rho}(-p_1+l)^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{(l-p_1)^2-m^2}\right] 2ie^2 g_{\mu\nu}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\nu\sigma}+(\xi-1)\frac{l^\nu l^\sigma}{l^2}\right)\right], \tag{4.20}$$

and the fourth one is



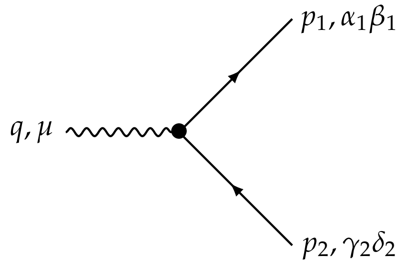
with an amplitude

$$\begin{aligned}
&-ie\Gamma^{\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2)_4 = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\alpha_2\beta_2\gamma_2\delta_2} \\
&+1_{\alpha_1\beta_1\gamma_2\delta_2}1_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\alpha_2\beta_2\gamma_2\delta_2} + \chi_{\alpha_1\beta_1\gamma_2\delta_2}\chi_{\alpha_2\beta_2\gamma_1\delta_1}) \\
&+\lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1}) \\
&+\lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1})]\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{(l)^2-m^2}\right] \\
&\times ie[T_{\rho\mu}(-l)^{\rho} - T_{\mu\rho}(l+p_1+p_2)^{\rho}]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{(l+p_1+p_2)^2-m^2}\right].
\end{aligned} \tag{4.21}$$

The divergent piece of the one-loop contribution can be written as

$$\begin{aligned}
&-ie\Gamma^{*\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2) = -i\left[\frac{e^3(\xi-3)}{16\pi^2\epsilon}\right][T_{\mu\rho}p_2^{\rho} - T_{\rho\mu}p_1^{\rho}]_{\alpha_1\beta_1\gamma_2\delta_2} \\
&-eg\left[\frac{e^2\left(g^2+2\right)+\lambda_1+\lambda_2+12\lambda_3}{16\pi^2\epsilon}\right](p_1^{\rho}+p_2^{\rho})(M_{\mu\rho})_{\alpha_1\beta_1\gamma_2\delta_2}.
\end{aligned} \tag{4.22}$$

The counterterm is



and its amplitude

$$\begin{aligned}
&-ie\Gamma^{\mu}_{\alpha_1\beta_1\gamma_2\delta_2}(-p_1-p_2,p_1,p_2)_{ct} = -ie\delta_e[T_{\mu\rho}p_2^{\rho} - T_{\rho\mu}p_1^{\rho}]_{\alpha_1\beta_1\gamma_2\delta_2} \\
&-eg\delta_{eg}(p_1^{\rho}+p_2^{\rho})(M_{\mu\rho})_{\alpha_1\beta_1\gamma_2\delta_2}.
\end{aligned} \tag{4.23}$$

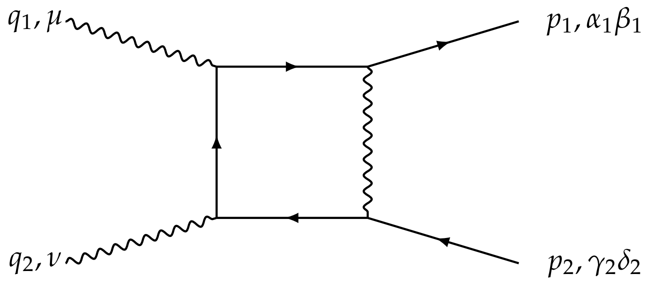
The amplitude of eq.(4.22) is canceled by the corresponding counterterm with

$$\delta_e = -\frac{e^2(\xi-3)}{16\pi^2\epsilon}, \tag{4.24}$$

$$\delta_{eg} = -\frac{e^2\left(g^2+2\right)+\lambda_1+\lambda_2+12\lambda_3}{16\pi^2\epsilon}. \tag{4.25}$$

4.5 TT$\gamma\gamma$ vertex

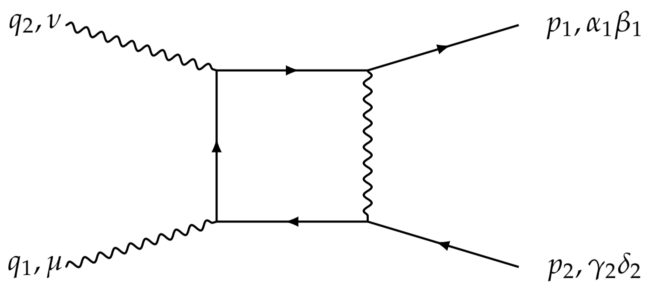
There are 12 diagrams contributing to the TT$\gamma\gamma$ vertex at one-loop.
The first one is given by



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,1} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_4\beta_4\gamma_4\delta_4}\left[\frac{i1^{\gamma_4\delta_4\alpha_2\beta_2}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\tau}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.26}$$

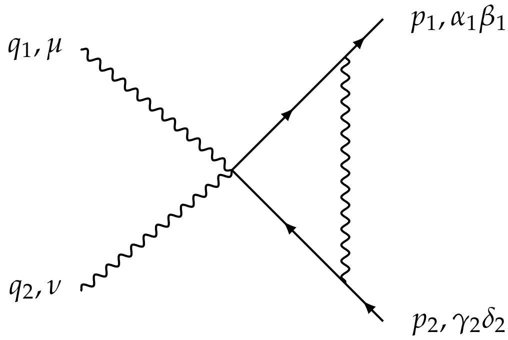
The second correction is given by



and its amplitude is

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,2} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_4\beta_4\gamma_4\delta_4}\left[\frac{i1^{\gamma_4\delta_4\alpha_2\beta_2}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\tau}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.27}$$

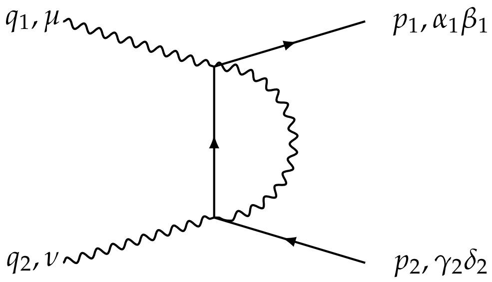
The third diagram is represented by



with an amplitude

$$
\begin{aligned}
i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,3} &= \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] \\
&\times 2ie^2 g_{\mu\nu}1_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right] \\
&\times ie[T_{\rho\tau}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right].
\end{aligned} \tag{4.28}
$$

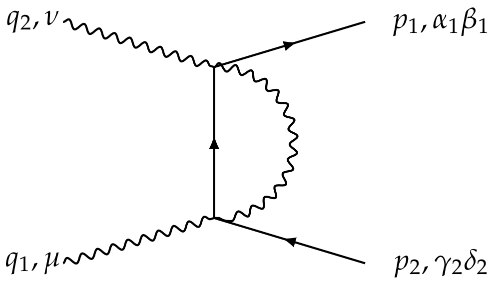
The fourth one is



and its amplitude

$$
\begin{aligned}
i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,4} &= \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma}1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \\
&\times 2ie^2 g_{\nu\tau}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right].
\end{aligned} \tag{4.29}
$$

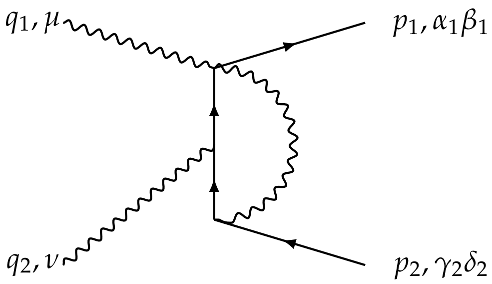
The fifth correction is given by



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,5} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\nu\sigma} 1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \times 2ie^2 g_{\mu\tau} 1_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.30}$$

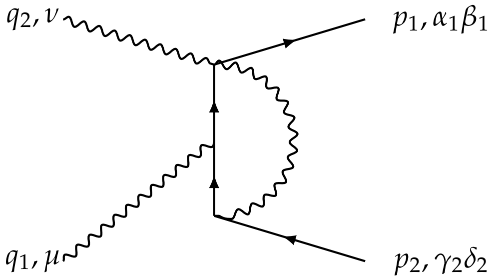
The sixth one is



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,6} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma} 1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] \times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right] \times ie[T_{\rho\tau}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.31}$$

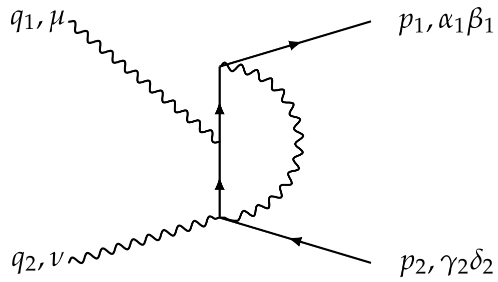
The seventh correction is given by



with amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,7} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\nu\sigma} 1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] \times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right] \times ie[T_{\rho\tau}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.32}$$

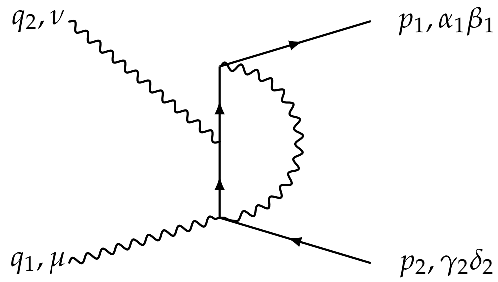
The eighth contribution is



and its amplitude is given by

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,8} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] \tag{4.33}$$
$$\times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right]$$
$$\times 2ie^2 g_{\nu\tau}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right].$$

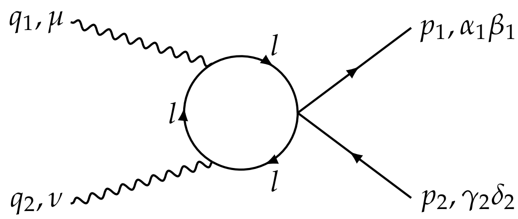
The ninth one is



and its amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,9} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] \tag{4.34}$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right]$$
$$\times 2ie^2 g_{\mu\tau}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right].$$

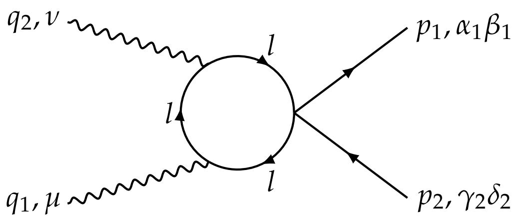
The tenth correction is given by



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,10} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\alpha_2\beta_2\gamma_2\delta_2} + 1_{\alpha_1\beta_1\gamma_2\delta_2}1_{\alpha_2\beta_2\gamma_1\delta_1}) \tag{4.35}$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\alpha_2\beta_2\gamma_2\delta_2} + \chi_{\alpha_1\beta_1\gamma_2\delta_2}\chi_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1})]$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_4\beta_4\gamma_4\delta_4}\left[\frac{i1^{\gamma_4\delta_4\alpha_2\beta_2}}{l^2-m^2}\right].$$

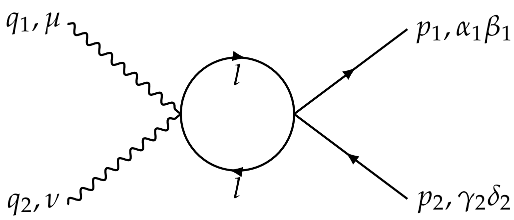
The eleventh one is given by



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,11} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\alpha_2\beta_2\gamma_2\delta_2} + 1_{\alpha_1\beta_1\gamma_2\delta_2}1_{\alpha_2\beta_2\gamma_1\delta_1}) \tag{4.36}$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\alpha_2\beta_2\gamma_2\delta_2} + \chi_{\alpha_1\beta_1\gamma_2\delta_2}\chi_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1})]$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_4\beta_4\gamma_4\delta_4}\left[\frac{i1^{\gamma_4\delta_4\alpha_2\beta_2}}{l^2-m^2}\right].$$

The twelfth diagram is



and its amplitude

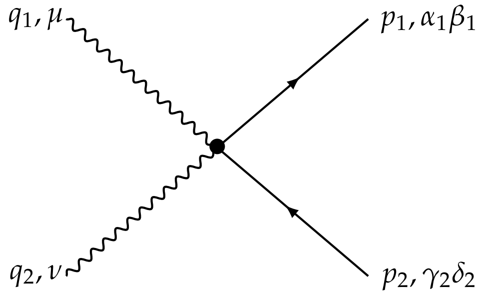
$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,12} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\alpha_2\beta_2\gamma_2\delta_2} + 1_{\alpha_1\beta_1\gamma_2\delta_2}1_{\alpha_2\beta_2\gamma_1\delta_1}) \tag{4.37}$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\alpha_2\beta_2\gamma_2\delta_2} + \chi_{\alpha_1\beta_1\gamma_2\delta_2}\chi_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1})]$$

$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_3\beta_3}}{l^2-m^2}\right] 2ie^2 g_{\mu\nu} 1_{\alpha_3\beta_3\gamma_3\delta_3} \left[\frac{i1^{\gamma_3\delta_3\alpha_2\beta_2}}{l^2-m^2}\right].$$

The corresponding divergent piece of the one-loop correction is

$$ie^2\Gamma^{*\mu\nu}_{\alpha\beta\gamma\delta} = ie^2\left[\frac{e^2(\xi-3)}{8\pi^2\epsilon}\right] 1_{\alpha\beta\gamma\delta} g^{\mu\nu}. \tag{4.38}$$

The counterterm is given by



with an amplitude

$$i\Gamma^{\mu\nu}_{\alpha_1\beta_1\gamma_2\delta_2\,ct} = 2ie^2\delta_{e2} 1_{\alpha_1\beta_1\gamma_2\delta_2} g_{\mu\nu}. \tag{4.39}$$

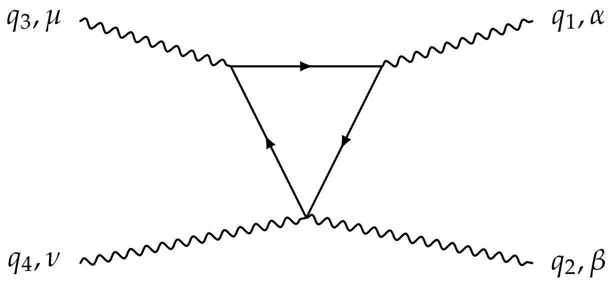
As anticipated from the Ward-Takahashi identities, the full TT$\gamma\gamma$ vertex becomes finite with δ_{e2} given by eq.(4.24).

$$\delta_{e2} = \delta_e = -\frac{e^2(\xi-3)}{16\pi^2\epsilon}. \tag{4.40}$$

4.6 $\gamma\gamma\gamma\gamma$ vertex

The one-loop correction to the $\gamma\gamma\gamma\gamma$ vertex involves 21 diagrams. It is important to point out that there are 9 diagrams obtained from diagrams $1-9$ reversing the arrow direction in the loop. Their amplitudes are denoted by a prime.

The first correction is given by



with an amplitude

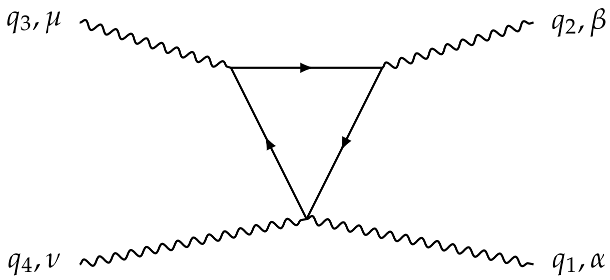
$$-i\mathcal{M}_{\alpha\beta\mu\nu\,1} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\alpha}(-l)^{\rho} - T_{\alpha\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.41}$$

$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$\times \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right] 2ie^2 g_{\nu\beta}1_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,1'} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\alpha}l^\rho - T_{\alpha\rho}(-l)^\rho]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.42}$$
$$\times \left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] 2ie^2 g_{\nu\beta}1_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}l^\rho - T_{\mu\rho}(-l)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2}\left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].$$

The correction number 2 is given by



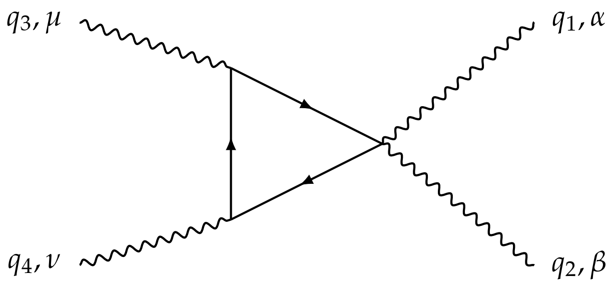
and its amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,2} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\beta}(-l)^\rho - T_{\beta\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.43}$$
$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$\times \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right] 2ie^2 g_{\nu\alpha}1_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,2'} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\beta}l^\rho - T_{\beta\rho}(-l)^\rho]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.44}$$
$$\times \left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] 2ie^2 g_{\nu\alpha}1_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}l^\rho - T_{\mu\rho}(-l)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2}\left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].$$

The number 3 is



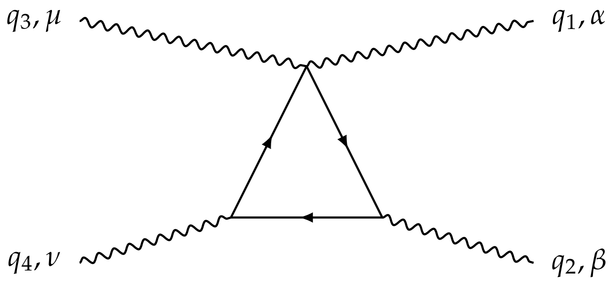
with an amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,3} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\beta\alpha} 1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \tag{4.45}$$
$$\times ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,3'} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\beta\alpha} 1_{\gamma_1\delta_1\alpha_1\beta_1}\left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] \tag{4.46}$$
$$\times ie[T_{\rho\nu}l^\rho - T_{\nu\rho}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}l^\rho - T_{\mu\rho}(-l)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2}\left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].$$

The diagram number 4 is represented by



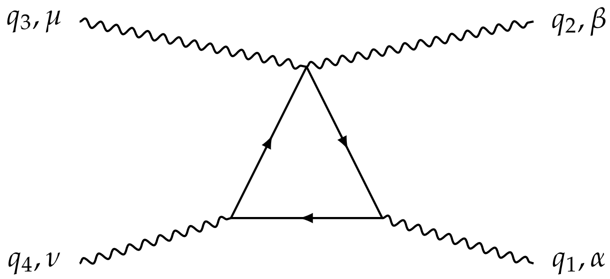
with amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,4} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\alpha\mu} 1_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \tag{4.47}$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\beta}(-l)^\rho - T_{\beta\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$\begin{aligned}-i\mathcal{M}_{\alpha\beta\mu\nu\,4'} = \mu^{2\varepsilon} &\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\alpha\mu} 1_{\gamma_1\delta_1\alpha_1\beta_1} \left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\beta}l^\rho - T_{\beta\rho}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3} \left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\nu}l^\rho - T_{\nu\rho}(-l)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2} \left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].\end{aligned} \tag{4.48}$$

The number 5 is given by



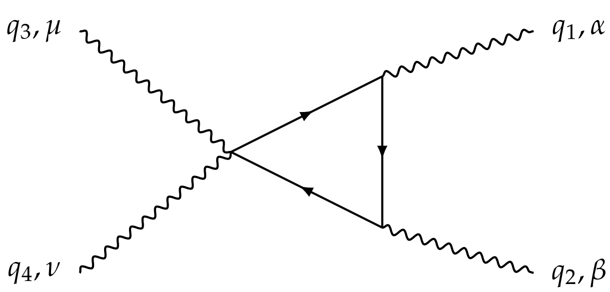
with an amplitude

$$\begin{aligned}-i\mathcal{M}_{\alpha\beta\mu\nu\,5} = \mu^{2\varepsilon} &\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\beta\mu} 1_{\alpha_1\beta_1\gamma_1\delta_1} \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2} \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\alpha}(-l)^\rho - T_{\alpha\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3} \left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],\end{aligned} \tag{4.49}$$

and

$$\begin{aligned}-i\mathcal{M}_{\alpha\beta\mu\nu\,5'} = \mu^{2\varepsilon} &\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\beta\mu} 1_{\gamma_1\delta_1\alpha_1\beta_1} \left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\alpha}l^\rho - T_{\alpha\rho}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3} \left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right] \\ &\times ie[T_{\rho\nu}l^\rho - T_{\nu\rho}(-l)^\rho]_{\gamma_2\delta_2\alpha_2\beta_2} \left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].\end{aligned} \tag{4.50}$$

The contribution number 6 is given by



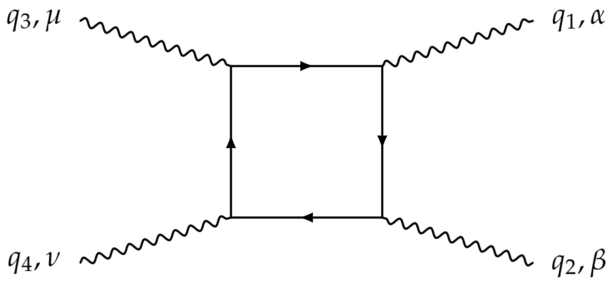
with an amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,6} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\alpha}(-l)^\rho - T_{\alpha\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.51}$$
$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] 2ie^2 g_{\mu\nu} 1_{\alpha_2\beta_2\gamma_2\delta_2} \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\beta}(-l)^\rho - T_{\beta\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3} \left[\frac{i1^{\gamma_3\delta_3\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,6'} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\alpha}l^\rho - T_{\alpha\rho}(-l)^\rho]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.52}$$
$$\times \left[\frac{i1^{\alpha_1\beta_1\gamma_3\delta_3}}{l^2-m^2}\right] ie[T_{\rho\beta}l^\rho - T_{\beta\rho}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3}$$
$$\times \left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right] 2ie^2 g_{\mu\nu} 1_{\gamma_2\delta_2\alpha_2\beta_2} \left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].$$

The number 7 is



with amplitude

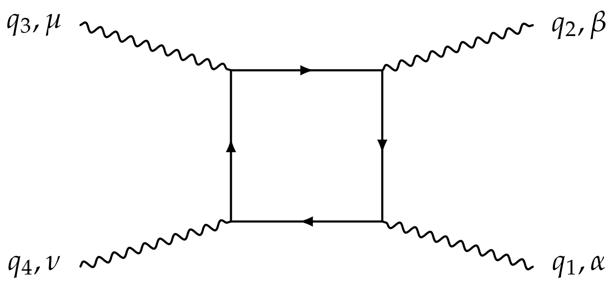
$$-i\mathcal{M}_{\alpha\beta\mu\nu\,7} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\alpha}(-l)^\rho - T_{\alpha\rho}l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.53}$$
$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\alpha_2\beta_2\gamma_2\delta_2} \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\alpha_3\beta_3\gamma_3\delta_3} \left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\beta}(-l)^\rho - T_{\beta\rho}l^\rho]_{\alpha_4\beta_4\gamma_4\delta_4} \left[\frac{i1^{\gamma_4\delta_4\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,7'} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\alpha}l^\rho - T_{\alpha\rho}(-l)^\rho]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.54}$$
$$\times \left[\frac{i1^{\alpha_1\beta_1\gamma_4\delta_4}}{l^2-m^2}\right] ie[T_{\rho\beta}l^\rho - T_{\beta\rho}(-l)^\rho]_{\gamma_4\delta_4\alpha_4\beta_4} \left[\frac{i1^{\alpha_4\beta_4\gamma_3\delta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}l^\rho - T_{\nu\rho}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3} \left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right]$$

$$ie[T_{\rho\mu}l^{\rho} - T_{\mu\rho}(-l)^{\rho}]_{\gamma_2\delta_2\alpha_2\beta_2} \times \left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2 - m^2}\right].$$

The number 8 is given by



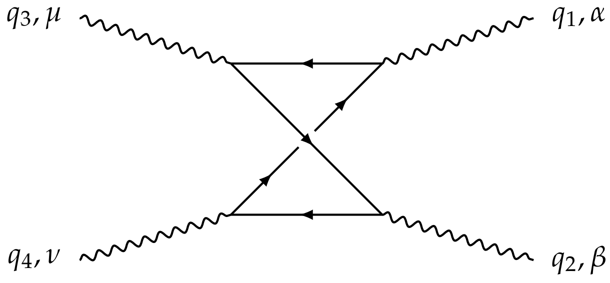
with an amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,8} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\beta}(-l)^{\rho} - T_{\beta\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.55}$$
$$\times \left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2 - m^2}\right] ie[T_{\rho\mu}(-l)^{\rho} - T_{\mu\rho}l^{\rho}]_{\alpha_2\beta_2\gamma_2\delta_2} \left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2 - m^2}\right]$$
$$\times ie[T_{\rho\nu}(-l)^{\rho} - T_{\nu\rho}l^{\rho}]_{\alpha_3\beta_3\gamma_3\delta_3} \left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2 - m^2}\right]$$
$$\times ie[T_{\rho\alpha}(-l)^{\rho} - T_{\alpha\rho}l^{\rho}]_{\alpha_4\beta_4\gamma_4\delta_4} \left[\frac{i1^{\gamma_4\delta_4\alpha_1\beta_1}}{l^2 - m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,8'} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[T_{\rho\beta}l^{\rho} - T_{\beta\rho}(-l)^{\rho}]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.56}$$
$$\times \left[\frac{i1^{\alpha_1\beta_1\gamma_4\delta_4}}{l^2 - m^2}\right] ie[T_{\rho\alpha}l^{\rho} - T_{\alpha\rho}(-l)^{\rho}]_{\gamma_4\delta_4\alpha_4\beta_4} \left[\frac{i1^{\alpha_4\beta_4\gamma_3\delta_3}}{l^2 - m^2}\right]$$
$$\times ie[T_{\rho\nu}l^{\rho} - T_{\nu\rho}(-l)^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3} \left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2 - m^2}\right]$$
$$ie[T_{\rho\mu}l^{\rho} - T_{\mu\rho}(-l)^{\rho}]_{\gamma_2\delta_2\alpha_2\beta_2} \times \left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2 - m^2}\right].$$

The correction number 9 is



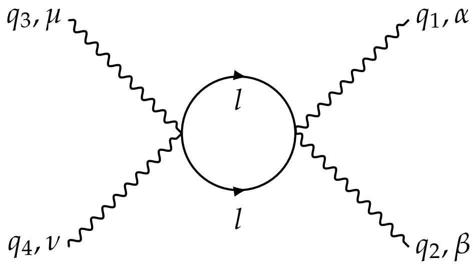
with amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,9} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\alpha}l^\rho - T_{\alpha\rho}(-l)^\rho]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] \tag{4.57}$$
$$\times ie[T_{\rho\nu}l^\rho - T_{\nu\rho}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}$$
$$\times\left[\frac{i1^{\gamma_2\delta_2\alpha_3\beta_3}}{l^2-m^2}\right] ie[T_{\rho\beta}l^\rho - T_{\beta\rho}(-l)^\rho]_{\alpha_3\beta_3\gamma_3\delta_3}\left[\frac{i1^{\gamma_3\delta_3\alpha_4\beta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\mu}l^\rho - T_{\mu\rho}(-l)^\rho]_{\alpha_4\beta_4\gamma_4\delta_4}\left[\frac{i1^{\gamma_4\delta_4\alpha_1\beta_1}}{l^2-m^2}\right],$$

and

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,9'} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[T_{\rho\alpha}(-l)^\rho - T_{\alpha\rho}l^\rho]_{\gamma_1\delta_1\alpha_1\beta_1} \tag{4.58}$$
$$\times\left[\frac{i1^{\alpha_1\beta_1\gamma_4\delta_4}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho - T_{\mu\rho}l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4}\left[\frac{i1^{\alpha_4\beta_4\gamma_3\delta_3}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\beta}(-l)^\rho - T_{\beta\rho}l^\rho]_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\alpha_3\beta_3\gamma_2\delta_2}}{l^2-m^2}\right]$$
$$ie[T_{\rho\nu}(-l)^\rho - T_{\nu\rho}l^\rho]_{\gamma_2\delta_2\alpha_2\beta_2}\times\left[\frac{i1^{\alpha_2\beta_2\gamma_1\delta_1}}{l^2-m^2}\right].$$

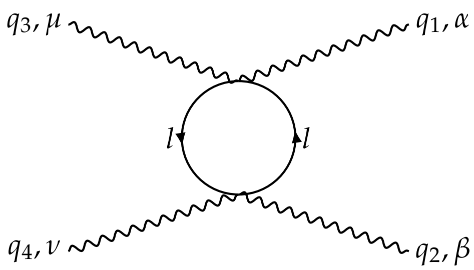
The correction number 10 is given by



with an amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,10} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] 2ie^2 g_{\alpha\beta}1_{\alpha_1\beta_1\gamma_1\delta_1} \tag{4.59}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right] 2ie^2 g_{\mu\nu}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{l^2-m^2}\right].$$

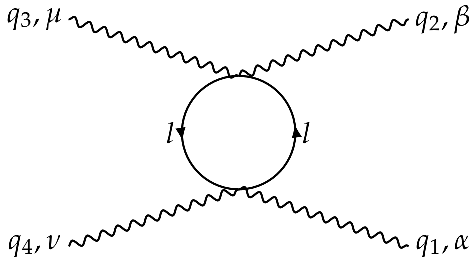
The number 11 is given by



with an amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,11} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right]2ie^2 g_{\alpha\mu}1_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right]2ie^2 g_{\nu\beta}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{l^2-m^2}\right]. \tag{4.60}$$

and the diagram number 12 is given by



with amplitude

$$-i\mathcal{M}_{\alpha\beta\mu\nu\,12} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right]2ie^2 g_{\beta\mu}1_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[\frac{i1^{\gamma_1\delta_1\alpha_2\beta_2}}{l^2-m^2}\right]2ie^2 g_{\nu\alpha}1_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\alpha_1\beta_1}}{l^2-m^2}\right]. \tag{4.61}$$

In this case, there is no counterterm available to cancel a potential divergence. Thus, if the model is renormalizable, the sum of all these pure QED diagrams must be finite. By an explicit calculation, we have found that the divergent piece of the total amplitude vanishes exactly.

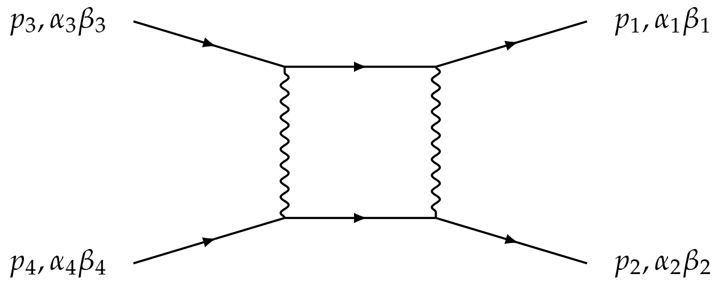
$$\begin{aligned}
-i\mathcal{M}_{\alpha\beta\mu\nu} \;=\; & -i\mathcal{M}_{\alpha\beta\mu\nu\,1} - i\mathcal{M}_{\alpha\beta\mu\nu\,1'} - i\mathcal{M}_{\alpha\beta\mu\nu\,2} - i\mathcal{M}_{\alpha\beta\mu\nu\,2'} - i\mathcal{M}_{\alpha\beta\mu\nu\,3} - i\mathcal{M}_{\alpha\beta\mu\nu\,3'} \\
& -i\mathcal{M}_{\alpha\beta\mu\nu\,4} - i\mathcal{M}_{\alpha\beta\mu\nu\,4'} - i\mathcal{M}_{\alpha\beta\mu\nu\,5} - i\mathcal{M}_{\alpha\beta\mu\nu\,5'} - i\mathcal{M}_{\alpha\beta\mu\nu\,6} - i\mathcal{M}_{\alpha\beta\mu\nu\,6'} \\
& -i\mathcal{M}_{\alpha\beta\mu\nu\,7} - i\mathcal{M}_{\alpha\beta\mu\nu\,7'} - i\mathcal{M}_{\alpha\beta\mu\nu\,8} - i\mathcal{M}_{\alpha\beta\mu\nu\,8'} - i\mathcal{M}_{\alpha\beta\mu\nu\,9} - i\mathcal{M}_{\alpha\beta\mu\nu\,9'} \\
& -i\mathcal{M}_{\alpha\beta\mu\nu\,10} - i\mathcal{M}_{\alpha\beta\mu\nu\,11} - i\mathcal{M}_{\alpha\beta\mu\nu\,12} = 0. \qquad (4.62)
\end{aligned}$$

Therefore, the four-gamma vertex function is finite, as expected.

4.7 TTTT vertex

The last potentially divergent function is the TTTT vertex and there are 19 diagrams contributing to the total amplitude.

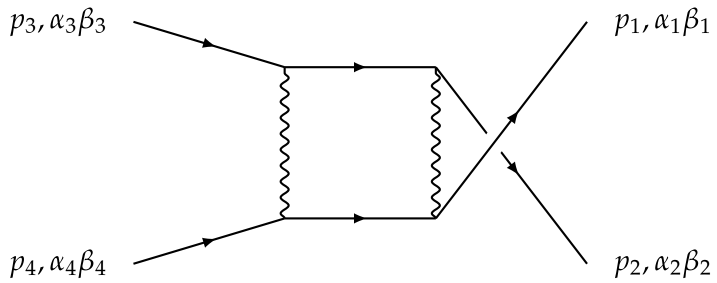
The first correction is given by



with an amplitude

$$\begin{aligned}
i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 1} = \mu^{2\varepsilon} \left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho} l^\rho]_{\alpha_1\beta_1\gamma_1\delta_1} \left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2 - m^2}\right] \\
\times ie[T_{\rho\mu}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3} \left[-\frac{i}{l^2}\left(g^{\nu\mu} + (\xi - 1)\frac{l^\nu l^\mu}{l^2}\right)\right] \\
\times ie[T_{\rho\nu} l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4} \left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2 - m^2}\right] \\
\times ie[-T_{\tau\rho}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2} \left[-\frac{i}{l^2}\left(g^{\sigma\tau} + (\xi - 1)\frac{l^\sigma l^\tau}{l^2}\right)\right].
\end{aligned} \tag{4.63}$$

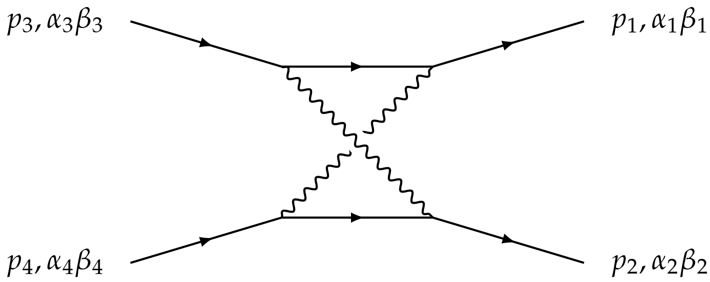
The second one is



and its amplitude is

$$\begin{aligned}
i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 2} = \mu^{2\varepsilon} \left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho} l^\rho]_{\alpha_2\beta_2\gamma_1\delta_1} \left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2 - m^2}\right] \\
\times ie[T_{\rho\mu}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3} \left[-\frac{i}{l^2}\left(g^{\nu\mu} + (\xi - 1)\frac{l^\nu l^\mu}{l^2}\right)\right] \\
\times ie[T_{\rho\nu} l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4} \left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2 - m^2}\right] \\
\times ie[-T_{\tau\rho}(-l)^\rho]_{\alpha_1\beta_1\gamma_2\delta_2} \left[-\frac{i}{l^2}\left(g^{\sigma\tau} + (\xi - 1)\frac{l^\sigma l^\tau}{l^2}\right)\right].
\end{aligned} \tag{4.64}$$

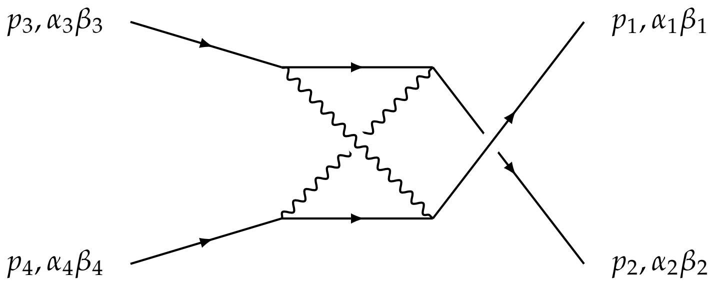
The third diagram is



with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,3} = \mu^{2\varepsilon}\left[\int\frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\ \times ie[T_{\rho\mu}(-l)^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\tau\mu}+(\xi-1)\frac{l^{\tau}l^{\mu}}{l^2}\right)\right] \\ \times ie[-T_{\tau\rho}l^{\rho}]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \\ \times ie[T_{\rho\nu}(-l)^{\rho}]_{\gamma_4\delta_4\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\nu}+(\xi-1)\frac{l^{\sigma}l^{\nu}}{l^2}\right)\right]. \tag{4.65}$$

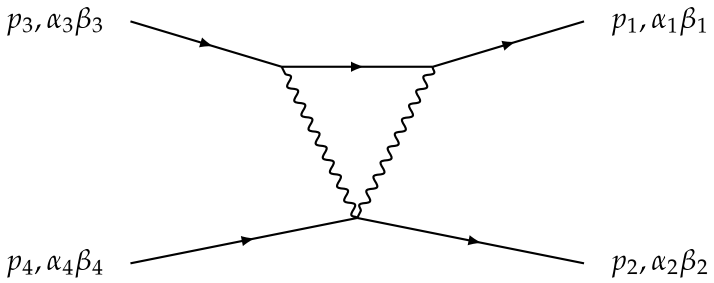
The fourth one is represented by



with amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,4} = \mu^{2\varepsilon}\left[\int\frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^{\rho}]_{\alpha_2\beta_2\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\ \times ie[T_{\rho\mu}(-l)^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\tau\mu}+(\xi-1)\frac{l^{\tau}l^{\mu}}{l^2}\right)\right] \\ \times ie[-T_{\tau\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \\ \times ie[T_{\rho\nu}(-l)^{\rho}]_{\gamma_4\delta_4\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\nu}+(\xi-1)\frac{l^{\sigma}l^{\nu}}{l^2}\right)\right]. \tag{4.66}$$

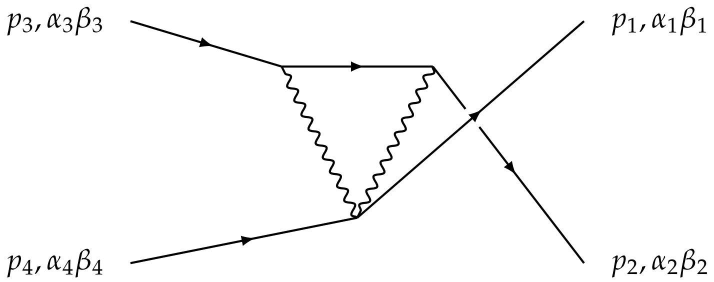
The fifth contribution is given by



with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 5} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \times ie[T_{\rho\mu}(-l)^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\nu\mu}+(\xi-1)\frac{l^{\nu}l^{\mu}}{l^2}\right)\right] \times 2ie^2 g_{\tau\nu}1_{\alpha_2\beta_2\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^{\sigma}l^{\tau}}{l^2}\right)\right]. \tag{4.67}$$

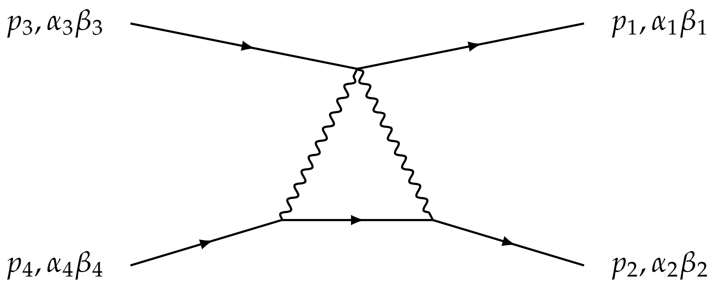
The sixth one is



and the amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 6} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\sigma\rho}l^{\rho}]_{\alpha_2\beta_2\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \times ie[T_{\rho\mu}(-l)^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\nu\mu}+(\xi-1)\frac{l^{\nu}l^{\mu}}{l^2}\right)\right] \times 2ie^2 g_{\tau\nu}1_{\alpha_1\beta_1\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^{\sigma}l^{\tau}}{l^2}\right)\right]. \tag{4.68}$$

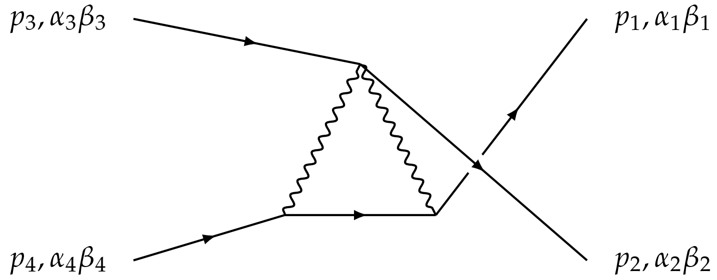
The seventh correction is



with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 7} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma} 1_{\alpha_1\beta_1\alpha_3\beta_3} \times\left[-\frac{i}{l^2}\left(g^{\nu\mu}+(\xi-1)\frac{l^\nu l^\mu}{l^2}\right)\right] ie[T_{\rho\mu}l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \times ie[-T_{\tau\rho}(-l)^\rho]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.69}$$

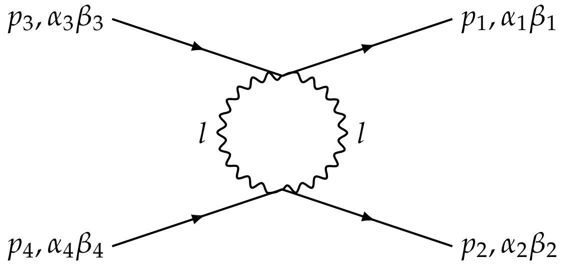
The eighth one is given by



with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 8} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma} 1_{\alpha_2\beta_2\alpha_3\beta_3} \times\left[-\frac{i}{l^2}\left(g^{\nu\mu}+(\xi-1)\frac{l^\nu l^\mu}{l^2}\right)\right] ie[T_{\rho\mu}l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \times ie[-T_{\tau\rho}(-l)^\rho]_{\alpha_1\beta_1\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.70}$$

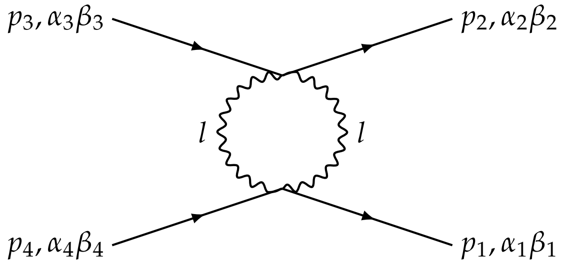
The ninth contribution is given by



and its amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4 9} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma} 1_{\alpha_1\beta_1\alpha_3\beta_3} \times\left[-\frac{i}{l^2}\left(g^{\nu\mu}+(\xi-1)\frac{l^\nu l^\mu}{l^2}\right)\right] 2ie^2 g_{\tau\nu} 1_{\alpha_2\beta_2\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\tau}+(\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right]. \tag{4.71}$$

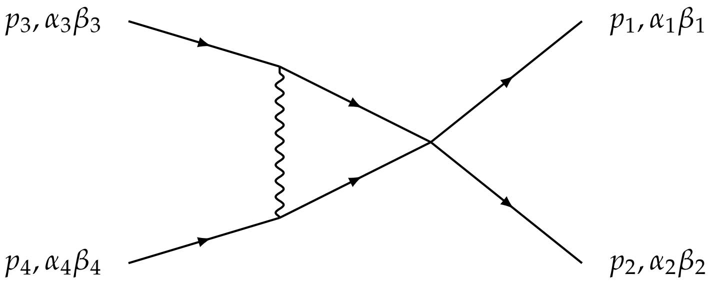
The tenth one is



with amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,10} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] 2ie^2 g_{\mu\sigma} 1_{\alpha_2\beta_2\alpha_3\beta_3} \tag{4.72}$$
$$\times\left[-\frac{i}{l^2}\left(g^{\nu\mu} + (\xi-1)\frac{l^\nu l^\mu}{l^2}\right)\right] 2ie^2 g_{\tau\nu} 1_{\alpha_1\beta_1\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\sigma\tau} + (\xi-1)\frac{l^\sigma l^\tau}{l^2}\right)\right].$$

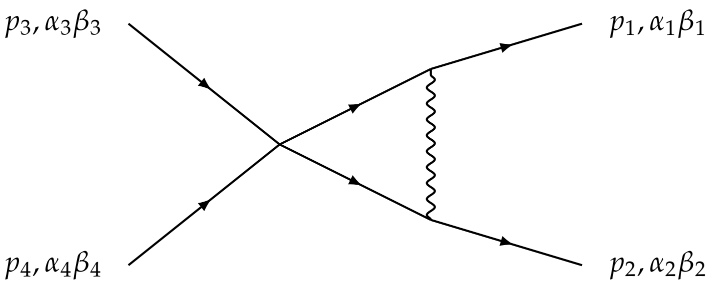
The eleventh diagram is represented by



and the amplitude

$$\begin{aligned}
&i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,11} = \mu^{2\epsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\alpha_2\beta_2\gamma_2\delta_2} + 1_{\alpha_1\beta_1\gamma_2\delta_2}1_{\alpha_2\beta_2\gamma_1\delta_1})\\
&+\lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\alpha_2\beta_2\gamma_2\delta_2} + \chi_{\alpha_1\beta_1\gamma_2\delta_2}\chi_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2}\\
&+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_2\delta_2} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_2\delta_2}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_1\delta_1})]\\
&\times\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] ie[T_{\rho\mu}(-l)^\rho]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\nu\mu} + (\xi-1)\frac{l^\nu l^\mu}{l^2}\right)\right]\\
&\times ie[T_{\rho\nu}l^\rho]_{\gamma_4\delta_4\alpha_4\beta_4}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right].
\end{aligned} \tag{4.73}$$

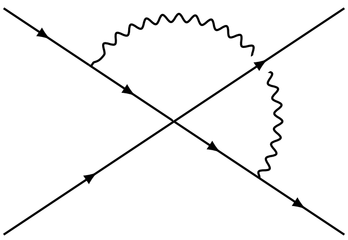
The twelfth one is



with amplitude

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,12} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\mu\rho}l^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\
& \times i[\lambda_1(1_{\gamma_3\delta_3\alpha_3\beta_3}1_{\gamma_4\delta_4\alpha_4\beta_4} + 1_{\gamma_3\delta_3\alpha_4\beta_4}1_{\gamma_4\delta_4\alpha_3\beta_3}) + \lambda_2(\chi_{\gamma_3\delta_3\alpha_3\beta_3}\chi_{\gamma_4\delta_4\alpha_4\beta_4} \\
& + \chi_{\gamma_3\delta_3\alpha_4\beta_4}\chi_{\gamma_4\delta_4\alpha_3\beta_3}) + \lambda_3(M^{\kappa\lambda}_{\gamma_3\delta_3\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_4\delta_4\alpha_4\beta_4} + M^{\kappa\lambda}_{\gamma_3\delta_3\alpha_4\beta_4}(M_{\kappa\lambda})_{\gamma_4\delta_4\alpha_3\beta_3}) \\
& + \lambda_4(S^{\kappa\lambda}_{\gamma_3\delta_3\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_4\delta_4\alpha_4\beta_4} + S^{\kappa\lambda}_{\gamma_3\delta_3\alpha_4\beta_4}(S_{\kappa\lambda})_{\gamma_4\delta_4\alpha_3\beta_3})]\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \\
& \times ie[-T_{\nu\rho}(-l)^{\rho}]_{\alpha_2\beta_2\gamma_2\delta_2}\left[-\frac{i}{l^2}\left(g^{\mu\nu} + (\xi-1)\frac{l^{\mu}l^{\nu}}{l^2}\right)\right]. \qquad (4.74)
\end{aligned}
$$

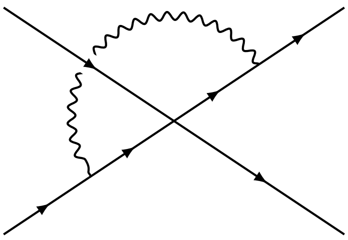
The thirteenth correction is given by



with an amplitude

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,13} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\mu\rho}(-l)^{\rho}]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\gamma_4\delta_4}}{l^2-m^2}\right] \\
& \times i[\lambda_1(1_{\alpha_1\beta_1\gamma_1\delta_1}1_{\gamma_4\delta_4\alpha_4\beta_4} + 1_{\alpha_1\beta_1\alpha_4\beta_4}1_{\gamma_4\delta_4\gamma_1\delta_1}) + \lambda_2(\chi_{\alpha_1\beta_1\gamma_1\delta_1}\chi_{\gamma_4\delta_4\alpha_4\beta_4} \\
& + \chi_{\alpha_1\beta_1\alpha_4\beta_4}\chi_{\gamma_4\delta_4\gamma_1\delta_1}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(M_{\kappa\lambda})_{\gamma_4\delta_4\alpha_4\beta_4} + M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\gamma_4\delta_4\gamma_1\delta_1}) \\
& + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_1\delta_1}(S_{\kappa\lambda})_{\gamma_4\delta_4\alpha_4\beta_4} + S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\gamma_4\delta_4\gamma_1\delta_1})]\left[\frac{i1^{\gamma_1\delta_1\gamma_3\delta_3}}{l^2-m^2}\right] \\
& \times ie[T_{\rho\nu}l^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[-\frac{i}{l^2}\left(g^{\mu\nu} + (\xi-1)\frac{l^{\mu}l^{\nu}}{l^2}\right)\right]. \qquad (4.75)
\end{aligned}
$$

The fourteenth one is

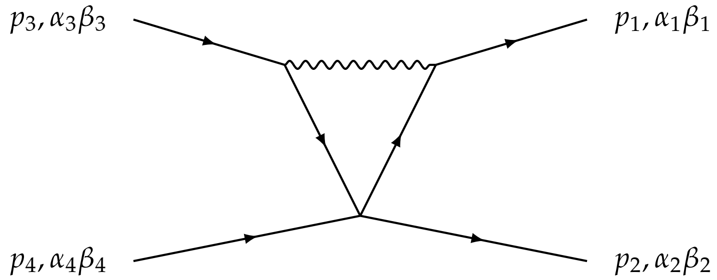


with an amplitude

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,14} = \mu^{2\varepsilon}\left[\int \frac{d^d l}{(2\pi)^d}\right] ie[-T_{\mu\rho}(-l)^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1}\left[\frac{i1^{\gamma_1\delta_1\gamma_2\delta_2}}{l^2-m^2}\right] \\
& \times i[\lambda_1(1_{\gamma_2\delta_2\alpha_3\beta_3}1_{\alpha_2\beta_2\gamma_3\delta_3} + 1_{\gamma_2\delta_2\gamma_3\delta_3}1_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_2(\chi_{\gamma_2\delta_2\alpha_3\beta_3}\chi_{\alpha_2\beta_2\gamma_3\delta_3}
\end{aligned}
$$

$$+\chi_{\gamma_2\delta_2\gamma_3\delta_3}\chi_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_3\beta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3} + M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3})$$
$$+\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_3\beta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3} + S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3})]\left[\frac{i1^{\gamma_3\delta_3\gamma_4\delta_4}}{l^2-m^2}\right]$$
$$\times ie[T_{\rho\nu}l^{\rho}]_{\gamma_4\delta_4\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\mu\nu}+(\xi-1)\frac{l^{\mu}l^{\nu}}{l^2}\right)\right]. \tag{4.76}$$

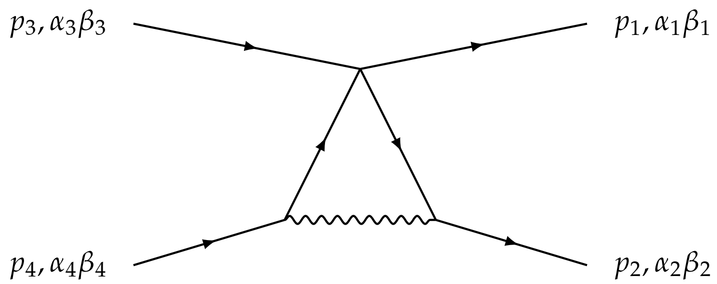
The fifteenth contribution is given by



and its amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,15} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] ie[-T_{\nu\rho}(-l)^{\rho}]_{\alpha_1\beta_1\gamma_1\delta_1}$$
$$\times\left[-\frac{i}{l^2}\left(g^{\mu\nu}+(\xi-1)\frac{l^{\mu}l^{\nu}}{l^2}\right)\right] ie[T_{\rho\mu}l^{\rho}]_{\gamma_3\delta_3\alpha_3\beta_3}\left[\frac{i1^{\gamma_4\delta_4\gamma_3\delta_3}}{l^2-m^2}\right]$$
$$\times i[\lambda_1(1_{\gamma_2\delta_2\gamma_4\delta_4}1_{\alpha_2\beta_2\alpha_4\beta_4} + 1_{\gamma_2\delta_2\alpha_4\beta_4}1_{\alpha_2\beta_2\gamma_4\delta_4}) + \lambda_2(\chi_{\gamma_2\delta_2\gamma_4\delta_4}\chi_{\alpha_2\beta_2\alpha_4\beta_4}$$
$$+\chi_{\gamma_2\delta_2\alpha_4\beta_4}\chi_{\alpha_2\beta_2\gamma_4\delta_4}) + \lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_4\delta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} + M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4})$$
$$+\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_4\delta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} + S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4})]\left[\frac{i1^{\gamma_1\delta_1\gamma_2\delta_2}}{l^2-m^2}\right]. \tag{4.77}$$

The sixteenth one is

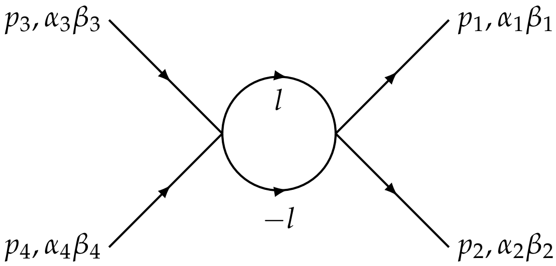


with an amplitude

$$i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,16} = \mu^{2\varepsilon}\left[\int\frac{d^dl}{(2\pi)^d}\right] i[\lambda_1(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\gamma_1\delta_1\gamma_3\delta_3} + 1_{\alpha_1\beta_1\gamma_3\delta_3}1_{\gamma_1\delta_1\alpha_3\beta_3})$$
$$+\lambda_2(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\gamma_1\delta_1\gamma_3\delta_3} + \chi_{\alpha_1\beta_1\gamma_3\delta_3}\chi_{\gamma_1\delta_1\alpha_3\beta_3}) + \lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\gamma_3\delta_3}$$
$$+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3}) + \lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\gamma_3\delta_3} + S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})]$$
$$\times\left[\frac{i1^{\gamma_3\delta_3\gamma_4\delta_4}}{l^2-m^2}\right] ie[T_{\rho\nu}(-l)^{\rho}]_{\gamma_4\delta_4\alpha_4\beta_4}\left[-\frac{i}{l^2}\left(g^{\mu\nu}+(\xi-1)\frac{l^{\mu}l^{\nu}}{l^2}\right)\right]$$

$$\times ie[-T_{\mu\rho}l^{\rho}]_{\alpha_2\beta_2\gamma_2\delta_2}\left[\frac{i1^{\gamma_2\delta_2\gamma_1\delta_1}}{l^2-m^2}\right]. \quad (4.78)$$

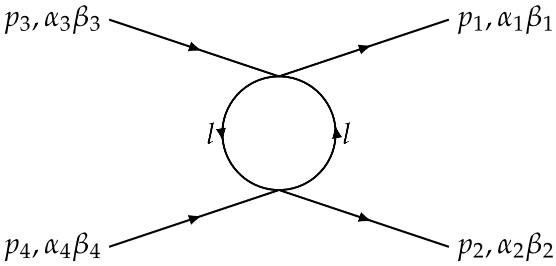
The seventeenth diagram is



and its amplitude is

$$\begin{aligned}
&i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,17} = \mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}i[\lambda_1(1_{\alpha_1\beta_1\gamma_3\delta_3}1_{\alpha_2\beta_2\gamma_4\delta_4}+1_{\alpha_1\beta_1\gamma_4\delta_4}1_{\alpha_2\beta_2\gamma_3\delta_3})\\
&+\lambda_2(\chi_{\alpha_1\beta_1\gamma_3\delta_3}\chi_{\alpha_2\beta_2\gamma_4\delta_4}+\chi_{\alpha_1\beta_1\gamma_4\delta_4}\chi_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4}\\
&+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_4\delta_4}+S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})]\\
&\times\left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right]i[\lambda_1(1_{\gamma_1\delta_1\alpha_3\beta_3}1_{\gamma_2\delta_2\alpha_4\beta_4}+1_{\gamma_1\delta_1\alpha_4\beta_4}1_{\gamma_2\delta_2\alpha_3\beta_3})+\lambda_2(\chi_{\gamma_1\delta_1\alpha_3\beta_3}\chi_{\gamma_2\delta_2\alpha_4\beta_4}\\
&+\chi_{\gamma_1\delta_1\alpha_4\beta_4}\chi_{\gamma_2\delta_2\alpha_3\beta_3})+\lambda_3(M^{\kappa\lambda}_{\gamma_1\delta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_2\delta_2\alpha_4\beta_4}+M^{\kappa\lambda}_{\gamma_1\delta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\gamma_2\delta_2\alpha_3\beta_3})\\
&+\lambda_4(S^{\kappa\lambda}_{\gamma_1\delta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_2\delta_2\alpha_4\beta_4}+S^{\kappa\lambda}_{\gamma_1\delta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\gamma_2\delta_2\alpha_3\beta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{l^2-m^2}\right]. \quad (4.79)
\end{aligned}$$

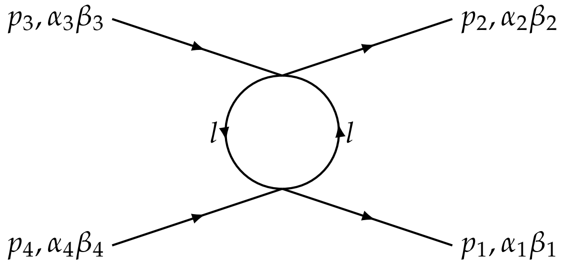
The eighteenth one is



and its amplitude

$$\begin{aligned}
&i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,18} = \mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}i[\lambda_1(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\gamma_1\delta_1\gamma_4\delta_4}+1_{\alpha_1\beta_1\gamma_4\delta_4}1_{\gamma_1\delta_1\alpha_3\beta_3})\\
&+\lambda_2(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\gamma_1\delta_1\gamma_4\delta_4}+\chi_{\alpha_1\beta_1\gamma_4\delta_4}\chi_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_3(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}\\
&+M^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(M_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})+\lambda_4(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4}+S^{\kappa\lambda}_{\alpha_1\beta_1\gamma_4\delta_4}(S_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})]\\
&\times\left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right]i[\lambda_1(1_{\gamma_2\delta_2\gamma_3\delta_3}1_{\alpha_2\beta_2\alpha_4\beta_4}+1_{\gamma_2\delta_2\alpha_4\beta_4}1_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_2(\chi_{\gamma_2\delta_2\gamma_3\delta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4}\\
&+\chi_{\gamma_2\delta_2\alpha_4\beta_4}\chi_{\alpha_2\beta_2\gamma_3\delta_3})+\lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}+M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})\\
&+\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4}+S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\gamma_3\delta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{l^2-m^2}\right]. \quad (4.80)
\end{aligned}$$

The nineteenth correction is given by



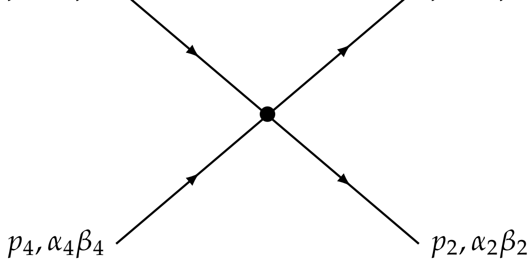
And its amplitude is

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,19} = \mu^{2\varepsilon}\int\frac{d^d l}{(2\pi)^d} i[\lambda_1(1_{\alpha_2\beta_2\alpha_3\beta_3}1_{\gamma_1\delta_1\gamma_4\delta_4} + 1_{\alpha_2\beta_2\gamma_4\delta_4}1_{\gamma_1\delta_1\alpha_3\beta_3}) \\
& +\lambda_2(\chi_{\alpha_2\beta_2\alpha_3\beta_3}\chi_{\gamma_1\delta_1\gamma_4\delta_4} + \chi_{\alpha_2\beta_2\gamma_4\delta_4}\chi_{\gamma_1\delta_1\alpha_3\beta_3}) + \lambda_3(M^{\kappa\lambda}_{\alpha_2\beta_2\alpha_3\beta_3}(M_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4} \\
& +M^{\kappa\lambda}_{\alpha_2\beta_2\gamma_4\delta_4}(M_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3}) + \lambda_4(S^{\kappa\lambda}_{\alpha_2\beta_2\alpha_3\beta_3}(S_{\kappa\lambda})_{\gamma_1\delta_1\gamma_4\delta_4} + S^{\kappa\lambda}_{\alpha_2\beta_2\gamma_4\delta_4}(S_{\kappa\lambda})_{\gamma_1\delta_1\alpha_3\beta_3})] \\
& \times\left[\frac{i1^{\gamma_3\delta_3\gamma_1\delta_1}}{l^2-m^2}\right] i[\lambda_1(1_{\gamma_2\delta_2\gamma_3\delta_3}1_{\alpha_1\beta_1\alpha_4\beta_4} + 1_{\gamma_2\delta_2\alpha_4\beta_4}1_{\alpha_1\beta_1\gamma_3\delta_3}) + \lambda_2(\chi_{\gamma_2\delta_2\gamma_3\delta_3}\chi_{\alpha_1\beta_1\alpha_4\beta_4} \\
& +\chi_{\gamma_2\delta_2\alpha_4\beta_4}\chi_{\alpha_1\beta_1\gamma_3\delta_3}) + \lambda_3(M^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(M_{\kappa\lambda})_{\alpha_1\beta_1\alpha_4\beta_4} + M^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_1\beta_1\gamma_3\delta_3}) \\
& +\lambda_4(S^{\kappa\lambda}_{\gamma_2\delta_2\gamma_3\delta_3}(S_{\kappa\lambda})_{\alpha_1\beta_1\alpha_4\beta_4} + S^{\kappa\lambda}_{\gamma_2\delta_2\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_1\beta_1\gamma_3\delta_3})]\left[\frac{i1^{\gamma_4\delta_4\gamma_2\delta_2}}{l^2-m^2}\right]. \qquad (4.81)
\end{aligned}
$$

The divergent part of the TTTT vertex is

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4} = \frac{1}{16\pi^2\epsilon}\Big\{e^4(3g^4-8g^2+6) + 2\lambda_1\left(e^2(2g^2+\xi)+\lambda_2+8\lambda_3+12\lambda_4\right) \\
& +11\lambda_1^2+3\lambda_2^2-8\lambda_2\lambda_4\Big\}(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\alpha_2\beta_2\alpha_4\beta_4} + 1_{\alpha_1\beta_1\alpha_4\beta_4}1_{\alpha_2\beta_2\alpha_3\beta_3}) \\
& +\frac{1}{8\pi^2\epsilon}\Big\{\lambda_2\left(e^2\left(2g^2+\xi\right)+4\lambda_1+8\lambda_3-8\lambda_4\right)+8\left(\lambda_3-\lambda_4\right)\left(e^2g^2+3\lambda_3-3\lambda_4\right) \\
& +4\lambda_2^2\Big\}(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4} + \chi_{\alpha_1\beta_1\alpha_4\beta_4}\chi_{\alpha_2\beta_2\alpha_3\beta_3}) - \frac{1}{16\pi^2\epsilon}\Big\{e^2g^2\left(\lambda_1+\lambda_2\right) \\
& +2\lambda_3\left(e^2\xi+4\lambda_1+4\lambda_2\right)+8\lambda_3^2-24\lambda_4^2\Big\}(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} + M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}) \\
& -\frac{1}{64\pi^2\epsilon}\Big\{e^4g^4+8\lambda_4\left(e^2\left(2g^2-\xi\right)-4\lambda_1+16\lambda_3-8\lambda_4\right)\Big\}(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} \\
& +S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}). \qquad (4.82)
\end{aligned}
$$

The counterterm is



and its amplitude

$$
\begin{aligned}
& i\Lambda_{\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\alpha_4\beta_4\,ct} = i[\lambda_1\delta_{\lambda 1}(1_{\alpha_1\beta_1\alpha_3\beta_3}1_{\alpha_2\beta_2\alpha_4\beta_4} + 1_{\alpha_1\beta_1\alpha_4\beta_4}1_{\alpha_2\beta_2\alpha_3\beta_3}) \\
& +\lambda_2\delta_{\lambda 2}(\chi_{\alpha_1\beta_1\alpha_3\beta_3}\chi_{\alpha_2\beta_2\alpha_4\beta_4} + \chi_{\alpha_1\beta_1\alpha_4\beta_4}\chi_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_3\delta_{\lambda 3}(M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} \\
& +M^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(M_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3}) + \lambda_4\delta_{\lambda 4}(S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_3\beta_3}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_4\beta_4} + S^{\kappa\lambda}_{\alpha_1\beta_1\alpha_4\beta_4}(S_{\kappa\lambda})_{\alpha_2\beta_2\alpha_3\beta_3})],
\end{aligned} \tag{4.83}
$$

and the corresponding counterterms that render the total amplitude finite are given in the MS scheme by

$$
\begin{aligned}
\delta_{\lambda 1} = & -\frac{1}{16\pi^2\lambda_1\epsilon}\Big\{e^4(3g^4 - 8g^2 + 6) + 2\lambda_1\left(e^2(2g^2 + \xi) + \lambda_2 + 8\lambda_3 + 12\lambda_4\right) \\
& +16\left(\lambda_4(e^2g^2 + 6\lambda_3) + \lambda_3(e^2g^2 + 3\lambda_3) + 6\lambda_4^2\right) \\
& +11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4\Big\},
\end{aligned} \tag{4.84}
$$

$$
\begin{aligned}
\delta_{\lambda 2} = & -\frac{1}{8\pi^2\lambda_2\epsilon}\Big\{\lambda_2\left(e^2\left(2g^2 + \xi\right) + 4\lambda_1 + 8\lambda_3 - 8\lambda_4\right) \\
& +8\left(\lambda_3 - \lambda_4\right)\left(e^2g^2 + 3\lambda_3 - 3\lambda_4\right) + 4\lambda_2^2\Big\},
\end{aligned} \tag{4.85}
$$

$$
\begin{aligned}
\delta_{\lambda 3} = & -\frac{1}{16\pi^2\lambda_3\epsilon}\Big\{e^2g^2\left(\lambda_1 + \lambda_2\right) + 2\lambda_3\left(e^2\xi + 4\lambda_1 + 4\lambda_2\right) \\
& +8\lambda_3^2 - 24\lambda_4^2\Big\},
\end{aligned} \tag{4.86}
$$

$$
\delta_{\lambda 4} = \frac{1}{64\pi^2\lambda_4\epsilon}\Big\{e^4g^4 + 8\lambda_4\left(e^2\left(2g^2 - \xi\right) - 4\lambda_1 + 16\lambda_3 - 8\lambda_4\right)\Big\}. \tag{4.87}
$$

4.8 Beta Functions

From the results obtained in eqs.(4.6,4.13,4.14,4.24,4.25,4.40,4.84,4.85,4.86,4.87) and the definition of the counterterms in eqs.(2.87,2.88), the relations between the bare and renormalized parameters of the theory are given by

$$
\begin{aligned}
e_0 = Z_1^{-\frac{1}{2}}Z_2^{-1}Z_e\mu^\epsilon e, \qquad e_0^2 = Z_1^{-1}Z_2^{-1}Z_{e2}\mu^{2\epsilon}e^2, \qquad \lambda_{0j} = Z_2^{-2}Z_{\lambda_j}\mu^{2\epsilon}\lambda_j, \\
g_0 = Z_e^{-1}Z_{eg}g, \qquad m_0^2 = Z_2^{-1}Z_m m^2,
\end{aligned} \tag{4.88}
$$

the renormalization constants defined in the MS scheme are

$$
Z_1^{\rm MS} = 1 + \frac{e^2(2g^2 - 1)}{8\pi^2\epsilon}, \tag{4.89}
$$

$$
Z_2^{\rm MS} = Z_{e2}^{\rm MS} = Z_e^{\rm MS} = 1 - \frac{e^2(\xi - 3)}{16\pi^2\epsilon}, \tag{4.90}
$$

$$
\begin{aligned}
Z_{\lambda_1}^{\rm MS} = 1 & - \frac{1}{16\pi^2\lambda_1\epsilon}\Big\{e^4(3g^4 - 8g^2 + 6) + 2\lambda_1\left(e^2(2g^2 + \xi) + \lambda_2 + 8\lambda_3 + 12\lambda_4\right) \\
& +16\left(\lambda_4(e^2g^2 + 6\lambda_3) + \lambda_3(e^2g^2 + 3\lambda_3) + 6\lambda_4^2\right) \\
& +11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4\Big\},
\end{aligned} \tag{4.91}
$$

$$
Z_{\lambda_2}^{\rm MS} = 1 - \frac{1}{8\pi^2\lambda_2\epsilon}\Big\{\lambda_2\left(e^2\left(2g^2 + \xi\right) + 4\lambda_1 + 8\lambda_3 - 8\lambda_4\right) \tag{4.92}
$$

$$+8\left(\lambda_3-\lambda_4\right)\left(e^2g^2+3\lambda_3-3\lambda_4\right)+4\lambda_2^2\Big\},$$

$$Z_{\lambda_3}^{\mathrm{MS}}=1-\frac{1}{16\pi^2\lambda_3\epsilon}\Big\{e^2g^2\left(\lambda_1+\lambda_2\right)+2\lambda_3\left(e^2\xi+4\lambda_1+4\lambda_2\right) \tag{4.93}$$
$$+8\lambda_3^2-24\lambda_4^2\Big\},$$

$$Z_{\lambda_4}^{\mathrm{MS}}=1+\frac{1}{64\pi^2\lambda_4\epsilon}\Big\{e^4g^4+8\lambda_4\left(e^2\left(2g^2-\xi\right)-4\lambda_1+16\lambda_3-8\lambda_4\right)\Big\}, \tag{4.94}$$

$$Z_{eg}^{\mathrm{MS}}=Z_e^{\overline{\mathrm{MS}}}+\delta_g^{\overline{\mathrm{MS}}}=1-\frac{1}{16\pi^2\epsilon}\Big\{e^2(g^2+\xi-1)+\lambda_1+\lambda_2+12\lambda_3\Big\}, \tag{4.95}$$

$$Z_m^{\mathrm{MS}}=Z_2^{\mathrm{MS}}+\delta_m^{\mathrm{MS}}=1-\frac{1}{16\pi^2\epsilon}\Big\{e^2\left(2g^2+\xi\right)+7\lambda_1+\lambda_2+8\lambda_3+12\lambda_4\Big\}. \tag{4.96}$$

According to these constants, the two different relations between e_0 and e in eq.(4.88) collapse to

$$e_0=Z_1^{-1/2}\mu^\epsilon e. \tag{4.97}$$

From eqs. (4.88-4.96) one can extract the following beta functions[2] $\beta_\eta\equiv\mu\frac{\partial\eta}{\partial\mu}$ and anomalous dimensions $\gamma_m\equiv\frac{\mu}{m}\frac{\partial m}{\partial\mu}$ in the $\epsilon\to 0$ limit:

$$\beta_e=\frac{1}{8\pi^2}\Big\{e^3\left(1-2g^2\right)\Big\}, \tag{4.98}$$

$$\beta_{eg}=-\frac{1}{8\pi^2}\Big\{g\left(e^2\left(g^2+2\right)+\lambda_1+\lambda_2+12\lambda_3\right)\Big\}, \tag{4.99}$$

$$\beta_{\lambda_1}=\frac{1}{8\pi^2}\Big\{e^4\left(-3g^4+8g^2-6\right)-2\lambda_1\left(e^2\left(2g^2+3\right)+\lambda_2+8\lambda_3+12\lambda_4\right) \tag{4.100}$$
$$-16\left(\lambda_4\left(e^2g^2+6\lambda_3\right)+\lambda_3\left(e^2g^2+3\lambda_3\right)+6\lambda_4^2\right)-11\lambda_1^2-3\lambda_2^2+8\lambda_2\lambda_4\Big\},$$

$$\beta_{\lambda_2}=-\frac{1}{4\pi^2}\Big\{\lambda_2\left(e^2\left(2g^2+3\right)+4\lambda_1+8\lambda_3-8\lambda_4\right) \tag{4.101}$$
$$+8\left(\lambda_3-\lambda_4\right)\left(e^2g^2+3\lambda_3-3\lambda_4\right)+4\lambda_2^2\Big\},$$

$$\beta_{\lambda_3}=-\frac{1}{8\pi^2}\Big\{e^2g^2\left(\lambda_1+\lambda_2\right)+2\lambda_3\left(3e^2+4\lambda_1+4\lambda_2\right)+8\lambda_3^2-24\lambda_4^2\Big\}, \tag{4.102}$$

$$\beta_{\lambda_4}=\frac{1}{32\pi^2}\Big\{e^4g^4+8\lambda_4\left(e^2\left(2g^2-3\right)-4\lambda_1+16\lambda_3-8\lambda_4\right)\Big\}, \tag{4.103}$$

$$\gamma_m=-\frac{1}{16\pi^2}\Big\{e^2\left(2g^2+3\right)+7\lambda_1+\lambda_2+8\lambda_3+12\lambda_4\Big\}. \tag{4.104}$$

4.9 Fixed points for β_{eg}

When a β function for some coupling in a quantum field theory has a zero at some value of the coupling parameter it is said that this is a fixed point. The scale-dependence of a theory is characterized by the way its coupling parameters depend on the energy-scale. This dependence is precisely described by the beta-functions of the theory. The importance of the fixed points is that they determine when a theory is scale-invariant. If we have a fixed point when the coupling parameter goes to zero (free field) we say that this is a trivial fixed point.

[2] The general procedure to determine the beta functions is presented in Appendix B.

In this section we want to analyze some fixed points for the function β_{eg}. They are particular cases for $\beta_{eg} = 0$.

First, turning off the electromagnetic interactions by taking $e = 0$ and $g = 0$, the theory reduces to the renormalizable model of pure self-interacting terms for the tensor field analyzed in Chapter 3:

$$\begin{aligned}
\beta_{\lambda 1} &= -\frac{1}{8\pi^2}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2 + 8\lambda_3 + 12\lambda_4\right) + 3\lambda_2^2 + 48\lambda_3^2 \\
&\quad -8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3 + \lambda_4\right)\Big\}, && (4.105)\\
\beta_{\lambda 2} &= -\frac{1}{\pi^2}\Big\{\lambda_2^2 + \lambda_1\lambda_2 + 2\lambda_2\left(\lambda_3 - \lambda_4\right) + 6\left(\lambda_3 - \lambda_4\right)^2\Big\}, && (4.106)\\
\beta_{\lambda 3} &= \frac{1}{\pi^2}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1 + \lambda_2 + \lambda_3\right)\Big\}, && (4.107)\\
\beta_{\lambda 4} &= -\frac{1}{\pi^2}\Big\{\lambda_4\left(\lambda_1 - 4\lambda_3 + 2\lambda_4\right)\Big\}, && (4.108)\\
\gamma_m &= -\frac{1}{16\pi^2}\Big\{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. && (4.109)
\end{aligned}$$

If we take $g = 0$ we get

$$\begin{aligned}
\beta_e &= \frac{e^3}{8\pi^2}, && (4.110)\\
\beta_{\lambda_1} &= -\frac{1}{8\pi^2}\Big\{6\left(e^4 + 8\lambda_3^2\right) + 2\lambda_1\left(3e^2 + \lambda_2 + 8\lambda_3 + 12\lambda_4\right) \\
&\quad +11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3 + \lambda_4\right)\Big\}, && (4.111)\\
\beta_{\lambda_2} &= -\frac{1}{4\pi^2}\Big\{\lambda_2\left(3e^2 + 4\lambda_1 + 8\lambda_3 - 8\lambda_4\right) + 4\lambda_2^2 + 24\left(\lambda_3 - \lambda_4\right)^2\Big\}, && (4.112)\\
\beta_{\lambda_3} &= \frac{1}{4\pi^2}\Big\{12\lambda_4^2 - \lambda_3\left(3e^2 + 4\lambda_1 + 4\lambda_2 + 4\lambda_3\right)\Big\}, && (4.113)\\
\beta_{\lambda_4} &= -\frac{1}{4\pi^2}\Big\{\lambda_4\left(3e^2 + 4\lambda_1 - 16\lambda_3 + 8\lambda_4\right)\Big\}, && (4.114)\\
\gamma_m &= -\frac{1}{16\pi^2}\Big\{3e^2 + 7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. && (4.115)
\end{aligned}$$

Taking $g = 0, \lambda_2 = 0$ we get

$$\begin{aligned}
\beta_e &= \frac{e^3}{8\pi^2}, && (4.116)\\
\beta_{\lambda_1} &= -\frac{1}{8\pi^2}\Big\{6\left(e^4 + 8\left(\lambda_3^2 + 2\lambda_4\lambda_3 + 2\lambda_4^2\right)\right) + 11\lambda_1^2 \\
&\quad +2\lambda_1\left(3\left(e^2 + 4\lambda_4\right) + 8\lambda_3\right)\Big\}, && (4.117)\\
\beta_{\lambda_2} &= -\frac{1}{\pi^2}\Big\{6\left(\lambda_3 - \lambda_4\right)^2\Big\}, && (4.118)\\
\beta_{\lambda_3} &= \frac{1}{4\pi^2}\Big\{12\lambda_4^2 - \lambda_3\left(3e^2 + 4\lambda_1 + 4\lambda_3\right)\Big\}, && (4.119)\\
\beta_{\lambda_4} &= -\frac{1}{4\pi^2}\Big\{\lambda_4\left(3e^2 + 4\lambda_1 - 16\lambda_3 + 8\lambda_4\right)\Big\}, && (4.120)
\end{aligned}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{3\left(e^2+4\lambda_4\right)+7\lambda_1+8\lambda_3\Big\}. \tag{4.121}$$

If we take $g=0, \lambda_3=0$ the beta functions become

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.122}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\Big\{6e^4+2\lambda_1\left(3\left(e^2+4\lambda_4\right)+\lambda_2\right)+11\lambda_1^2 \tag{4.123}$$

$$+3\lambda_2^2+96\lambda_4^2-8\lambda_2\lambda_4\Big\},$$

$$\beta_{\lambda_2} = -\frac{1}{4\pi^2}\Big\{\lambda_2\left(3e^2+4\lambda_1-8\lambda_4\right)+4\lambda_2^2+24\lambda_4^2\Big\}, \tag{4.124}$$

$$\beta_{\lambda_3} = \frac{3\lambda_4^2}{\pi^2}, \tag{4.125}$$

$$\beta_{\lambda_4} = -\frac{1}{4\pi^2}\Big\{\lambda_4\left(3e^2+4\lambda_1+8\lambda_4\right)\Big\}, \tag{4.126}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{3\left(e^2+4\lambda_4\right)+7\lambda_1+\lambda_2\Big\}. \tag{4.127}$$

Taking $g=0, \lambda_4=0$ we get

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.128}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\Big\{6\left(e^4+8\lambda_3^2\right)+2\lambda_1\left(3e^2+\lambda_2+8\lambda_3\right)+11\lambda_1^2+3\lambda_2^2\Big\}, \tag{4.129}$$

$$\beta_{\lambda_2} = -\frac{1}{4\pi^2}\Big\{\lambda_2\left(3e^2+4\lambda_1+8\lambda_3\right)+4\lambda_2^2+24\lambda_3^2\Big\}, \tag{4.130}$$

$$\beta_{\lambda_3} = -\frac{1}{4\pi^2}\Big\{\lambda_3\left(3e^2+4\lambda_1+4\lambda_2+4\lambda_3\right)\Big\}, \tag{4.131}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{3e^2+7\lambda_1+\lambda_2+8\lambda_3\Big\}. \tag{4.132}$$

If we take $g=0, \lambda_2=0, \lambda_3=0$ it is obtained

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.133}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\Big\{6e^4+6\lambda_1\left(e^2+4\lambda_4\right)+11\lambda_1^2+96\lambda_4^2\Big\}, \tag{4.134}$$

$$\beta_{\lambda_2} = -\frac{6\lambda_4^2}{\pi^2}, \tag{4.135}$$

$$\beta_{\lambda_3} = \frac{3\lambda_4^2}{\pi^2}, \tag{4.136}$$

$$\beta_{\lambda_4} = -\frac{1}{4\pi^2}\Big\{\lambda_4\left(3e^2+4\lambda_1+8\lambda_4\right)\Big\}, \tag{4.137}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{3\left(e^2+4\lambda_4\right)+7\lambda_1\Big\}. \tag{4.138}$$

If $g = 0, \lambda_2 = 0, \lambda_4 = 0$ we get

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.139}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\left\{6\left(e^4 + 8\lambda_3^2\right) + 2\lambda_1\left(3e^2 + 8\lambda_3\right) + 11\lambda_1^2\right\}, \tag{4.140}$$

$$\beta_{\lambda_2} = -\frac{6\lambda_3^2}{\pi^2}, \tag{4.141}$$

$$\beta_{\lambda_3} = -\frac{1}{4\pi^2}\left\{\lambda_3\left(3e^2 + 4\lambda_1 + 4\lambda_3\right)\right\}, \tag{4.142}$$

$$\gamma_m = -\frac{1}{16\pi^2}\left\{\left(3e^2 + 7\lambda_1 + 8\lambda_3\right)\right\}. \tag{4.143}$$

With $g = 0, \lambda_3 = 0, \lambda_4 = 0$ we get

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.144}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\left\{6e^4 + 2\lambda_1\left(3e^2 + \lambda_2\right) + 11\lambda_1^2 + 3\lambda_2^2\right\}, \tag{4.145}$$

$$\beta_{\lambda_2} = -\frac{1}{4\pi^2}\left\{\lambda_2\left(3e^2 + 4\lambda_1 + 4\lambda_2\right)\right\}, \tag{4.146}$$

$$\gamma_m = -\frac{1}{16\pi^2}\left\{3e^2 + 7\lambda_1 + \lambda_2\right\}. \tag{4.147}$$

Finally, taking $g = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$ we get

$$\beta_e = \frac{e^3}{8\pi^2}, \tag{4.148}$$

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\left\{6e^4 + 6e^2\lambda_1 + 11\lambda_1^2\right\}, \tag{4.149}$$

$$\gamma_m = -\frac{1}{16\pi^2}\left\{3e^2 + 7\lambda_1\right\}. \tag{4.150}$$

This trivial fixed point for the beta functions of the theory corresponds to the limit in which each component of the tensor $B^{\mu\nu}$ behaves as a complex scalar field in a $\lambda\phi^4$ theory with $\lambda_1 = -\lambda/2$ because the information about the spin of the field disappears when $g = 0$.

Now we analyze the case without self-interactions. The function β_{eg} reads

$$\beta_{eg} = -\frac{1}{8\pi^2}\left\{g\left(e^2\left(g^2 + 2\right)\right)\right\}. \tag{4.151}$$

Then, we can see that there is no non-trivial finite value of the gyromagnetic factor that gives a fixed point in the absence of self-interactions.

Conclusions

In this work, we have studied the one-loop renormalization of a model describing a spin-1 matter field in the $(1,0) \oplus (0,1)$ representation of the HLG in the Poincaré projector formalism. The kinetic part of the Lagrangian is of Klein-Gordon type and the spin information is encoded by a Pauli-like term and the four independent quartic self-interactions that can be built from the covariant basis for this representation space, given by the complete set of tensors presented in [4]. The analysis has been carried out in an arbitrary covariant gauge, with arbitrary gyromagnetic factor and including all the independent parity conserving self-interactions.

Throughout the document we analyzed two models. We first discussed a simplified model in which only the self-interacting terms were considered. For this particular case, was shown that it is renormalizable and the β functions were also determined.

After studying this first case, we analyzed the Quantum Electrodynamics of the complete model. We also showed that the $\gamma\gamma\gamma$ vertex is null, verifying in this way the charge conjugation invariance of the theory. In our model the $\gamma\gamma\gamma\gamma$ vertex is finite because the divergent amplitudes that appear in the contributions to this vertex they all cancel.

Moreover, some of the obtained counterterms are equal. This is related to the Ward-Takahashi identities and it verifies that the model is gauge invariant. We prove that the model is renormalizable at one-loop order. Moreover, the β functions related to the parameters of the Lagrangian were determined and we presented some fixed points for β_{eg}. The most important of these points are the cases when $g = 0, e = 0$ because it recovers the functions of the simplified model studied in Chapter 3 and when $g = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$ because it describes scalar QED. On the other hand, the β_{λ_i} are all nonzero for any non-vanishing real value of the gyromagnetic factor g, even if all self-interactions are set to $\lambda_i = 0, i = 1, \ldots, 4$. This means that, oppositely to the spin $1/2$ case [22], pure electrodynamics for matter fields of spin 1 is not viable for $g \neq 0$, as self interactions are necessary to make the theory renormalizable.

The main conclusion of the work is that the theory is renormalizable for any value of the gyromagnetic factor, displaying a rich set of renormalization group equations. In contrast to the analogous spin $1/2$ case studied in [22] where it was demonstrated that there exists a fixed point for $g = 2$, there is no non-trivial finite value for the gyromagnetic factor that allows the existence of a pure electrodynamics without the inclusion of self-interactions.

Appendix A

General procedure to determine the Feynman rules

The action is given by the expression

$$\Gamma_0[\phi_a] = \int d^4x \mathcal{L}(\phi_a). \tag{A.1}$$

Let's define the functional derivative

$$\frac{\delta\phi_a(p)}{\delta\phi_b(q)} = \delta^a_b \delta^4(p-q). \tag{A.2}$$

Specifically for the field $B^{\alpha\beta}$

$$\frac{\delta B^{\alpha\beta}(p)}{\delta B^{\gamma\delta}(q)} = 1^{\alpha\beta}_{\gamma\delta}\delta^4(p-q) = \frac{1}{2}(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma)\delta^4(p-q). \tag{A.3}$$

For the field A^μ can also be written

$$\frac{\delta A^\mu(p)}{\delta A^\nu(q)} = \delta^\mu_\nu \delta^4(p-q). \tag{A.4}$$

Now let's define the n-points function

$$i\Gamma^n_{a_1,a_2,...,a_n}(p_1,p_2,...,p_n)(2\pi)^4\delta^4(p_1+p_2+...+p_n) = i(2\pi)^{4n}\frac{\delta^n\Gamma_0[\phi_a]}{\delta\phi_{a_1}(p_1)\delta\phi_{a_2}(p_2)...\delta\phi_{a_n}(p_n)}. \tag{A.5}$$

The n-points functions are the Green's functions that result from the perturbative analysis in Quantum Field Theory. They are defined as the functional expectation value of the product of n field operators at different positions

$$\Gamma^n(x_1,x_2,\ldots,x_n) = \langle 0|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|0\rangle \tag{A.6}$$

Propagators

A propagator is a Green's function that gives the probability amplitude of a particle displacing between two points. A propagator is related to the 2 points function as

$$\Delta_{a_1,a_2} = i[\Gamma_{0\,a_1,a_2}(p,-p)]^{-1}. \tag{A.7}$$

Vertices

The vertex in momenta space is then given by the rule

$$i\Gamma^{n}_{0\,a_1,a_2,\ldots,a_n}. \tag{A.8}$$

Appendix B

General procedure to determine the beta functions

The physics of the model must be independent of the arbitrary scale μ introduced in the dimensional regularization. To determine the beta functions we have to use the relations given in eq. (4.88).

In general, these relations have the form

$$\alpha_{i\,0} = Z^a_{\alpha_i}\alpha_i, \tag{B.1}$$

where α_i stands for e, g, λ_j.

The first step is to determine the natural logarithm of the parameters

$$\ln\alpha_i = a\ln Z_{\alpha_i} + \ln\alpha_i. \tag{B.2}$$

Next, we have to compute the derivative with respect to the natural logarithm of μ

$$\frac{\partial\ln\alpha_{i\,0}}{\partial\ln\mu} = a\frac{\partial\ln Z_{\alpha_i}}{\partial\ln\mu} + \frac{\partial\ln\alpha_i}{\partial\ln\mu} = 0. \tag{B.3}$$

Now

$$\frac{\partial\ln\alpha_i}{\partial\ln\mu} = \frac{\mu}{\alpha_i}\frac{\partial\alpha_i}{\partial\mu}. \tag{B.4}$$

Defining the β functions

$$\beta_{\alpha_i} \equiv \frac{\partial\alpha_i}{\partial\ln\mu} = \mu\frac{\partial\alpha_i}{\partial\mu}, \tag{B.5}$$

we can write

$$\frac{\partial\ln\alpha_i}{\partial\ln\mu} = \frac{\beta_{\alpha_i}}{\alpha_i}. \tag{B.6}$$

The $Z_{\alpha_i}s$ can be written as

$$Z_{\alpha_i} = 1 + X_{\alpha_i}, \tag{B.7}$$

and its natural logarithm as

$$\ln Z_{\alpha_i} = X_{\alpha_i} = Z_{\alpha_i} - 1. \tag{B.8}$$

Then we can write

$$\frac{\partial \ln Z_{\alpha_i}}{\partial \ln \mu} = \mu \frac{\partial (Z_{\alpha_i} - 1)}{\partial \mu} = \sum_i \mu \frac{\partial (Z_{\alpha_i} - 1)}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial \mu} = \sum_i \beta_{\alpha_i} \frac{\partial (Z_{\alpha_i} - 1)}{\partial \alpha_i}. \tag{B.9}$$

Then eq.(B.3) becomes

$$\frac{\partial \ln \alpha_{i\,0}}{\partial \ln \mu} = \sum_i \beta_{\alpha_i} \left(a \frac{\partial (Z_{\alpha_i} - 1)}{\partial \alpha_i} \right) + \frac{\beta_{\alpha_i}}{\alpha_i} = 0. \tag{B.10}$$

With this general description we compute all the beta functions and anomalous dimensions.

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Universidad de Guanajuato

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P R E S E N T E

Estimado Doctor David Delepine,

Por este medio le informo que he leído y revisado la tesis de maestría del estudiante Lic.Ailier Rivero Acosta. El trabajo se titula: **"Renormalization of a model for spin-1 matter fields"**, y el asesor es el Dr. Carlos Vaquera Araujo.

He hecho al estudiante recomendaciones para el documento final, hemos discutido sobre el trabajo y he constatado que posee un gran dominio del tema de tesis. Además de que los resultados por él obtenidos, son de relevancia y originalidad científica. Me complace informarle que estoy de acuerdo con que se realice la presentación del trabajo de tesis, puesto que el mismo cuenta con sobrados requisitos para la obtención del grado de Maestría en Física.

ATENTAMENTE
"LA VERDAD OS HARÁ LIBRES"

Dra. Nana Geraldine Cabo Bizet
Departamento de Física
División de Ciencias e Ingenierías
Universidad de Guanajuato

León Guanajuato, a 21 de agosto de 2020

Dr. David Delepine
Director de la División de Ciencias e Ingenierías
Campus León, Universidad de Guanajuato
PRESENTE

Estimado Dr. Delepine

Me permito informarle que he leído y revisado la tesis titulada **"Renormalization of a model for spin-1 matter fields".** Dicha tesis la realizó el estudiante **Ailier Rivero Acosta**, como requisito para obtener el grado de Maestro en Física.

Considero que el trabajo hecho por Ailier es notable, incluso los resultados han sido ya publicados en una revista internacional. Dicho lo anterior, extiendo mi aval para que su trabajo sea defendido en un examen profesional.

Sin más que agregar, agradezco su atención y aprovecho la ocasión para enviarle un cordial saludo.

ATENTAMENTE
"LA VERDAD OS HARÁ LIBRES"

Dr. Juan Barranco Monarca
Departamento de Física
DCI, Campus León

León, Guanajuato, 26 de Agosto de 2020

Dr. David Delepine

Director DCI

Presente

Por este conducto le informo que he leído la tesis titulada "***Renormalization of a model for spin-1 matter fields***" que para obtener el grado de Maestro en Física ha formulado el C. Ailier Rivero Acosta.

En mi opinión, este trabajo reúne las características de calidad y forma para el grado al que se aspira, por lo cual, no tengo inconveniente en que se realice la defensa de tesis correspondiente.

Sin otro particular, reciba un cordial saludo.

Atentamente

Dr. Mauro Napsuciale Mendivil

Profesor Titular C

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DEPARTAMENTO DE FISICA, DIVISION DE CIENCIAS E INGENIERÍAS, CAMPUS LEÓN

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León, Guanajuato, Agosto 28, 2020

Dr. David Delepine
Director
División de Ciencias e Ingenierías
PRESENTE

Por medio de la presente me permito informar que he leído la tesis titulada **"Renormalization of a model for spin-1 matter fields"**, que para obtener el grado de **Maestría en Física** ha sido elaborada por el **Lic. Ailier Rivero Acosta**. En mi opinión, la tesis cumple con los requisitos de calidad correspondientes al grado académico al que se aspira. Las correcciones sugeridas por mi parte han sido atendidas, por lo cual recomiendo se proceda a la defensa de la tesis.

Atentamente

Dr. Carlos Alberto Vaquera Araujo
Cátedra Conacyt
Departamento de Física DCI-UG
DCI, Campus León