Automated Computation of the Instantaneous Velocity Centers of Planar Linkages (Determinación Automatizada de los Centros Instantáneos de Velocidad de Mecanismos Planos).

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Abstract (Resumen)

This document presents a summary of the activities carried out during the XXVI Science Summer (XXVI Verano de la Ciencia). Since the group had an international student, from India, whose native language is English, it was decided to submit at least some of the documents in English. The project included two related but different topics: The automated computation of the instantaneous velocity centers of planar determinate linkages, and the determination of the instantaneous velocity centers of two-degree of freedom planar linkages. Both topics are related to the topics included in reasonable courses on planar kinematics but are usually beyond the topics usually included in those courses. Furthermore, the topics are of enough current interest to produce results that can be published in international scholarly journals.

Keywords (Palabras clave): Kinematics, Instantaneous velocity centers, Computer Aided Analysis.

Common background

In this section, the definitions, notations, and background that are common to the two topics of the project will be included in this section. Unless explicitly said in contrary, it will always be assumed that the linkage is a planar linkage, where X-Y is the plane of motion, therefore all the rotation axes are parallel to the Z axis.

An **instantaneous velocity center (IVC) or instantaneous rotation center** in a planar linkage is defined as two coincident points that belong to different links such that:

1. One of the links moves with respect to the other with a rotation around an axis parallel to the Z axis that passes through the coincident points.
2. Both points have the same velocity with respect to another reference frame.
3. The relative velocity between these two coincident points is zero.

It can be proved that these three definitions are equivalent.

An **instantaneous velocity center** can be classified according with different criteria:

1. According with the procedure for locating the centers:
   a. **Primary**. When the instantaneous center is located by using only the definition of an instantaneous velocity center.
   b. **Secondary**. When the instantaneous center is located by using the Aronhold-Kennedy theorem or another method such as the one used for the determination of indeterminate linkages ---a linkage is denoted as indeterminate if it is impossible to find all the secondary IVCs using only the classical Aronhold-Kennedy approach--- or the method presented here for the determination of the instantaneous velocity centers of two-degree of freedom linkages.
2. According if the relative motion between the links is absolute or relative. In other words, if one of the links is the fixed link or not.
   a. **Absolute**. If one of the two links involved in the instantaneous center is the fixed links.
   b. **Relative**. If none of the two links involved in the instantaneous center is the fixed links.

The notation for the instantaneous velocity center is as follows. The instantaneous velocity center O_{ij}, represents the instantaneous velocity center associated with the links i and j. It will be assumed that i > j.

Computation of the instantaneous velocity centers of two-degrees of freedom planar linkages

It is quite common that an exhausted professor looking for project topics, almost at the end of a semester, chooses, without giving proper thinking, the determination of the instantaneous velocity centers of a two-degree of freedom planar linkage as a project topic ---the last author passed through a very similar experience that did not involve a project. It is then, when the professor realizes that the task, although has some similarities and common background that the instantaneous velocity centers of one-degree of freedom linkages, it also has many differences that makes the endeavor an interesting but non-trivial task.

Consider a simple five-links planar linkage with only revolute pairs shown in Figure 1. Using Grübler's criterion it is straightforward to prove that it has 2 degrees of freedom (DOF), in addition it has 10 instantaneous centers.

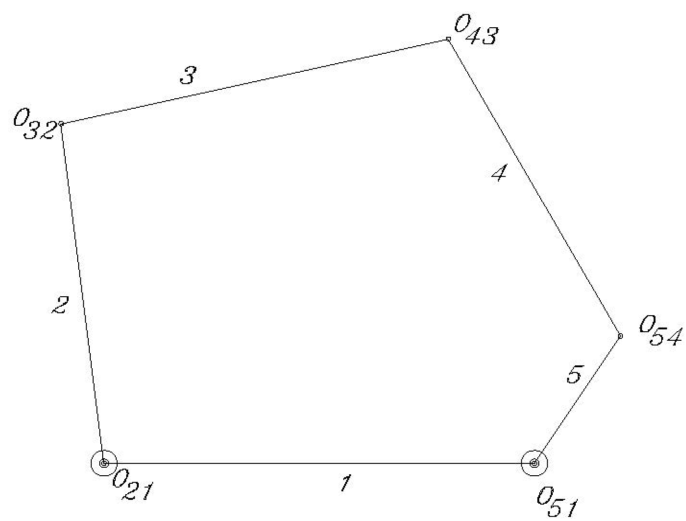

Figure 1. Planar five-bar linkage with 2 degrees of freedom.

From the 10 instantaneous velocity centers, 5 of them are primary. They are associated with the linkage's revolute joints. They are O_{21}, O_{32}, O_{43}, O_{54}, and O_{51}. The remaining center O_{31}, O_{41}, O_{42}, O_{52}, and O_{53} are all secondary centers.

At this point, the commonalities between instantaneous centers of one-DOF linkages and of two or more DOF linkages end. For linkages with one DOF, the location of the instantaneous velocity centers is a geometric property; namely, it only depends on the position of the linkage. In fact, for the location of the instantaneous velocity centers is not necessary to know the velocity of the input link. Moreover, in the location process it is unnecessary to know which link is the input link. In two-DOF linkages, the location of the secondary centers is not, in general, a geometric property, but it is a kinematic property. Namely, in addition to know the position of the linkage it is necessary to know in advance the input links and their velocities, angular or linear. Then you can imagine the confusion of a student when the final project becomes a topic that goes far beyond the knowledge acquired in a regular kinematics of machinery course.

It is important to recognize that in the determination of secondary IVCs of a two-DOF linkage there still exists a geometric property. Namely, the location of any secondary IVCs will always be located along a line called the **center line** associated with the secondary IVC. One of the important goals of the project is to be able to determine these center lines for arbitrary two-DOF planar linkages.

During the activities of the summer, the location of two-DOF linkages were obtained following two approaches that allow the user to verify the results of one another. Both approaches require the selection of the input links. For the linkage of figure 1, the input links were 2 and 5, both links are connected to the fixed link (1), and it was assumed that the input velocities, that in this case are also absolute angular velocities are ω_2 and ω_5. However, it is unnecessary to assign any numeric values to them.

The first approach consists in solving the velocity analysis of the linkage, for that purpose, the velocity equation, using very basic formulae of Rigid-Body Dynamics were employed, and from the result of the velocity analysis it was possible to find the symbolic equation of the secondary IVCs and the location of the velocity centers when:

1. The angular velocity $\omega_2 = 0$, and

2. The angular velocity ω_5 =0.

The location of these two points determines the center line of the corresponding secondary IVC.

The second approach is rather simple, if it is assumed that the angular velocity ω_2 =0, the two-DOF linkage becomes a one-DOF linkage, where the classical Aronhold-Kennedy approach can be used to find the secondary IVCs obtained in the first approach when velocity ω_2 =0. Similarly, if it is assumed that the angular velocity ω_5 =0, the two-DOF linkage becomes a one-DOF linkage, where the classical Aronhold-Kennedy approach can be used to find the secondary IVCs obtained in the first approach when velocity ω_5 =0. To pursue this process, it was used a simple program for automated drawing.

Using these two IVCs the center line associated with the IVC will be found.

Figure 2, shows the graphical computations when it is assumed that the angular velocity ω_2 =0, therefore link 2 can be assumed to be rigidly connected to the fixed link. The only two secondary centers are also indicated. The remaining three secondary IVCs can be found using the condition that link 2 can be regarded as rigidly attached to the fixed link 1.

Figure 3, shows the graphical computations when it is assumed that the angular velocity ω_5 =0, therefore link 5 can be assumed to be rigidly connected to the fixed link. The only two secondary centers are also indicated, but their location exceeds the dimensions of the picture. The remaining three secondary IVCs can be found using the condition that link 5 can be regarded as rigidly attached to the fixed link 1.

Finally, Figure 4 presents the center lines of the 5 secondary IVCs of the five-bar 2-DOF shown in Figure 1. These center lines pass through the IVCs centers determined and verified using both approaches outlined above. These are the only geometrical properties of secondary IVCs in a two-DOF planar linkage. These centerlines are named according to the corresponding instantaneous velocity center. Namely C_{31} for O_{31}, C_{41} for O_{41}, C_{42} for O_{42}, C_{52} for O_{52}, and C_{53} for O_{53}.

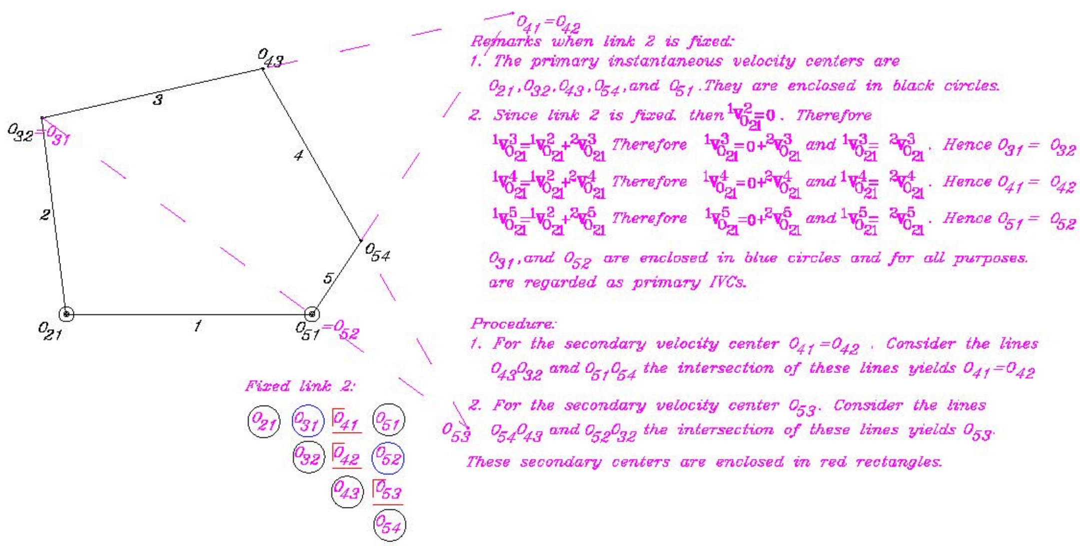

Figure 2. Determination of the secondary IVCs when ω_2= 0.

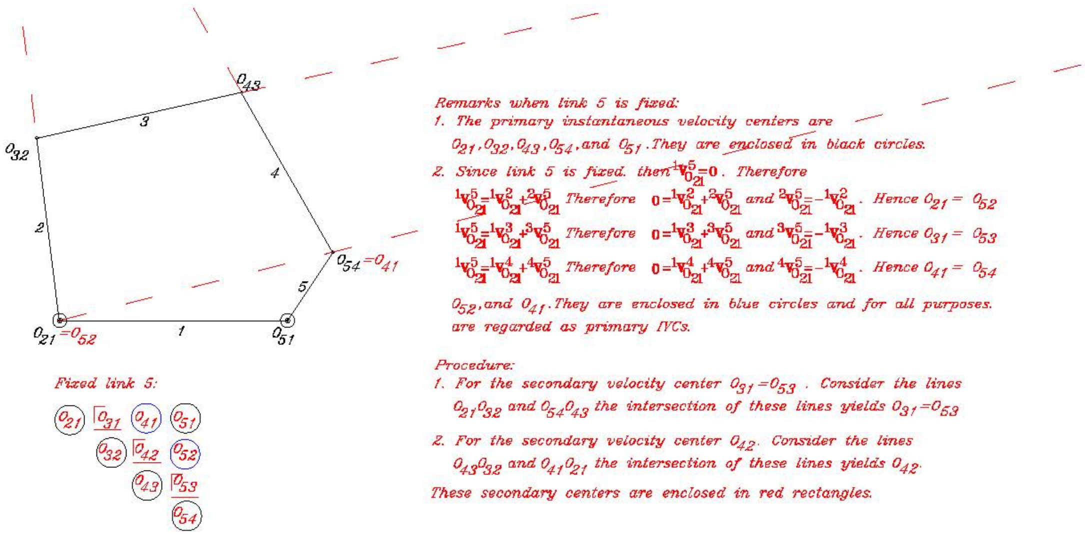

Figure 3. Determination of the secondary IVCs when $\omega_5 = 0$.

These centerlines have been the subject of interest of relatively recent scholarly papers. The approach was also successfully proven with a seven-bar planar linkage with a prismatic pair.

Automated computation of the instantaneous velocity centers of one-degree of freedom planar determinate linkages

The second part of the research focused on the automated computation of the instantaneous velocity centers of one-degree of freedom determinate planar linkages. As previously indicated a linkage is denoted determinate if all its secondary IVCs can be found using the Aronhold-Kennedy approach.

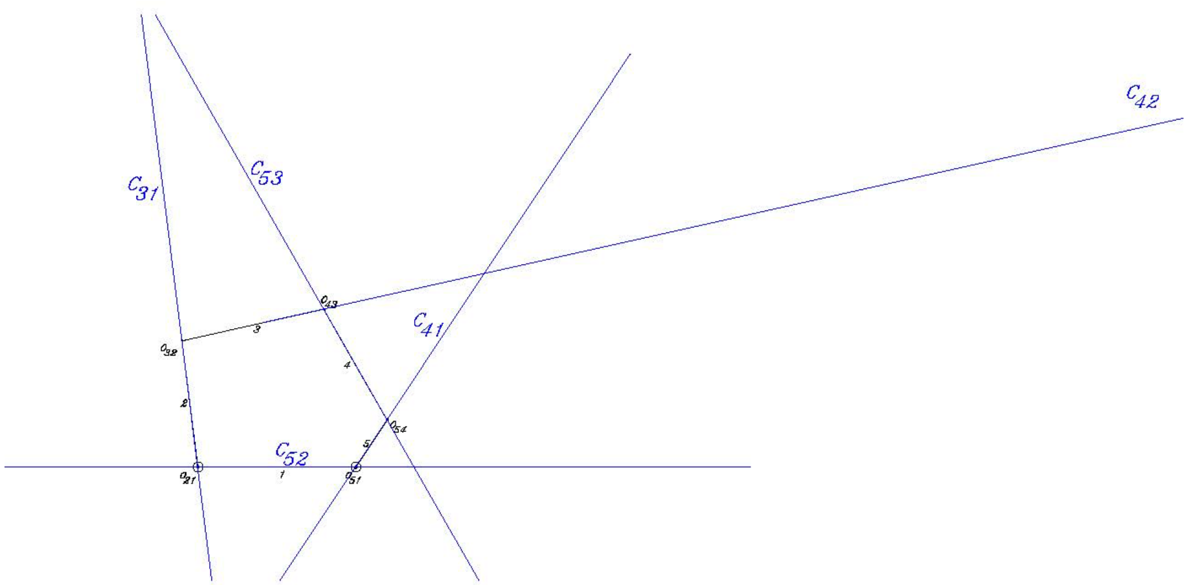

Figure 4. Determination of the center lines associated with the secondary IVCs of the five-bar linkage shown in Figure 1.

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This topic goes back to the second part of the XIX century until the middle of the XX century. During those dates, the IVC determination using a graphical approach was one of the few techniques available to engineers due to the lack of computers. Obviously, after the development of the digital computer, the velocity and acceleration analysis of planar and spatial linkages can be rather easily carried out without the need of the IVCs. However, the close relationship between IVCs and instantaneous screw axis, a fundamental concept in Screw Theory extensively used in spatial kinematics and robotics, has kept the topic an interesting one.

One of the objectives of the automation process of finding the IVCs of a planar linkage is that the process should be contained in one application. In other words, it was required that the application employed for the computation of the IVCs must be the same employed for drawing the analysis aids and the linkage with the primary and secondary instantaneous velocity centers. For that purpose, it was employed a symbolic mathematics program.

The linkage chosen to illustrate the automated process is shown in Figure 5. The linkage has one-DOF and 15 instantaneous velocity centers, 7 are primary centers given by O_{21}, O_{32}, O_{43}, O_{41}, O_{53}, O_{64}, and O_{65}. The remaining 8 centers are secondary ones. They are given by O_{31}, O_{42}, O_{51}, O_{52}, O_{54}, O_{61}, O_{62}, and O_{63}. The location of these secondary centers is the focus of this part of the research activities during the research summer.

Figure 6 shows the initial drawings aids indicating the primary IVCs enclosed in dark purple circles and the corresponding secants in the big circle located in the right part of the figure.

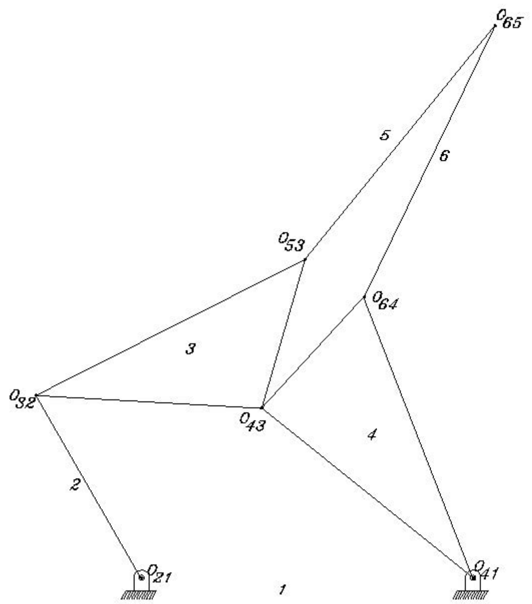

Figure 5. Planar linkage used to illustrate the process of automated determination of the IVCs.

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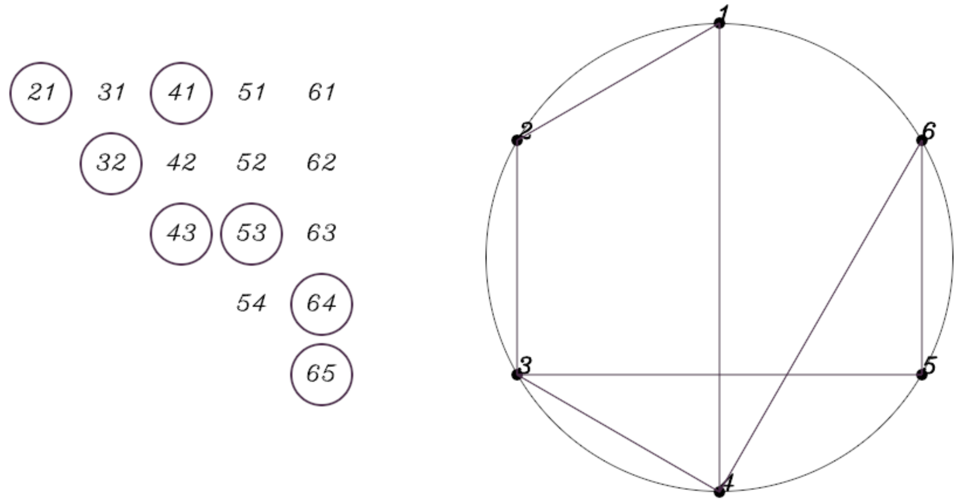

Figure 6. Initial stage of the drawing aids for the linkage shown in Figure 5. Only the primary IVCs are shown, and the corresponding secant is draw in the circle at the right.

Figure 7 shows, in addition, the secondary instantaneous velocity centers obtained during the first iteration of the program. These newly found IVCs are enclosed in blue circles and the corresponding secants are also drawn, in the circle at the right part of the figure also in blue.

Figure 8 shows the final stage of the computation, it includes the secondary instantaneous velocity centers obtained during the second iteration of the program. These newly found IVCs are enclosed in pink circles and the corresponding secants are also drawn, in the circle at the right part of the figure also in pink.

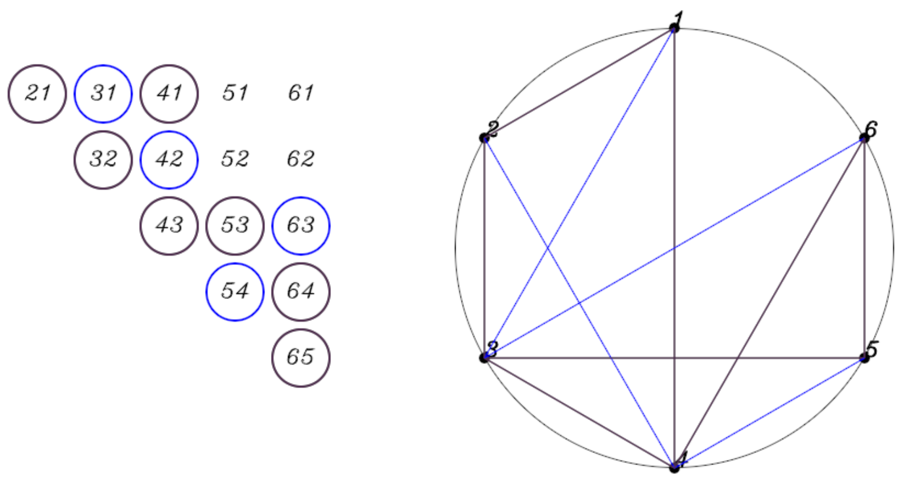

Figure 7. Intermediate stage of the drawing aids for the linkage shown in Figure 5. The secondary IVCs found in the first iteration of the program are also included.

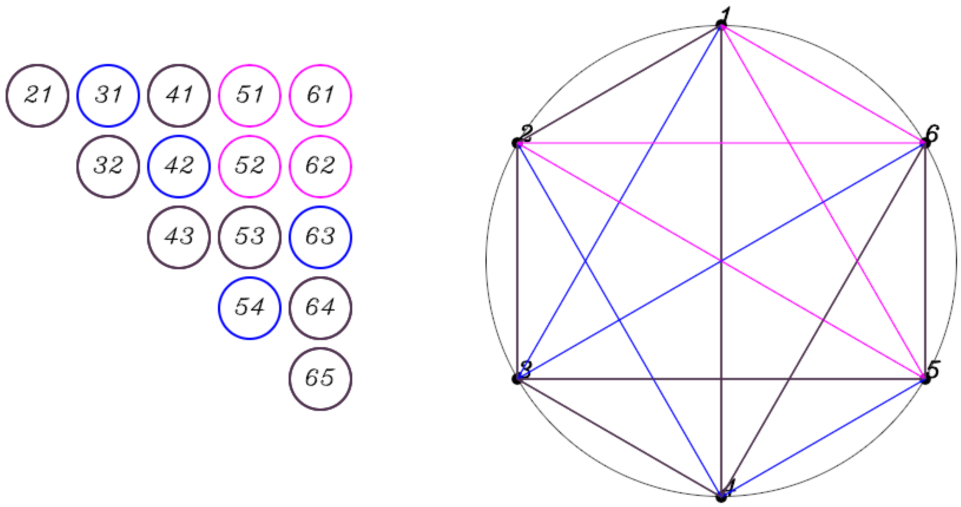

Figure 8. Final stage of the drawing aids for the linkage shown in Figure 5. The secondary IVCs found in the first and second iterations of the program are also included.

Figure 9 shows the graphical results, the figure contains the original linkage and the primary instantaneous velocity centers in black. Figure 9 also includes the secondary instantaneous velocity centers obtained in the first iteration of the program, together with the lines whose intersection determines the center, all drawn in blue. Finally, Figure 9 includes the secondary instantaneous velocity centers obtained during the second iteration of the program, together with the lines whose intersection determines the centers all drawn in pink.

The results were obtained in a few seconds compared with the one or two hours necessary for a skilled draftsman to complete the tasks. In addition, the use of a symbolic mathematics program ensures that the results have no error if the original coordinates of the primary centers are integer or quotient numbers and minimal error otherwise. The same can not be said even for a skilled draftsman. Of course, these computations can be carried also using automated drafting programs which override some of the concerns indicated here.

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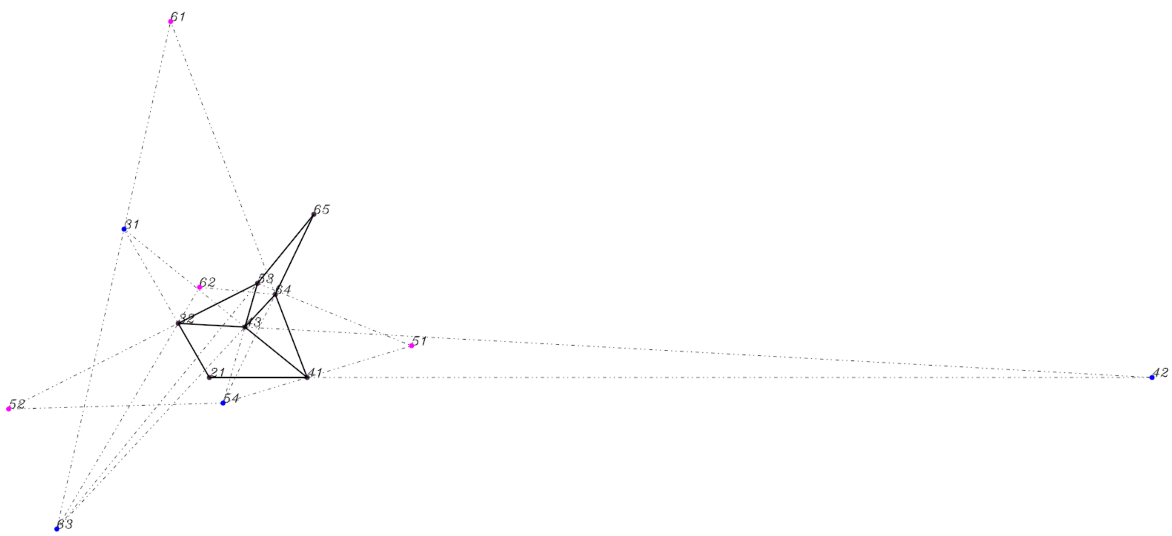

Figure 9. The results indicating the original linkage, the primary IVCs in black, the IVCs obtained in the first iteration in blue and the IVCs obtained in the second iteration in pink. The lines required to graphically obtain by intersection the secondary IVCs are also shown.

Further extensions

During this research, it was considered to extend the results to the automated determination of the instantaneous rotation axes of spherical linkages, the first result of this approach is shown in Figure 10.

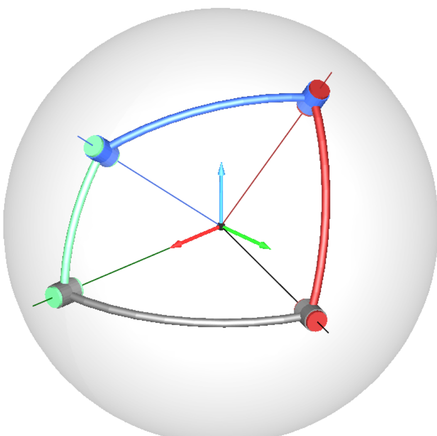

Figure 10. Drawing of a four-bar spherical linkage using the symbolic mathematics program.

Conclusions (Conclusiones)

The results obtained during the activities of the summer program fulfilled the expectations at the definition of the project. In the following months, the work group will polish the results and start writing the drafts for the papers to be submitted to scholarly journals. Some results are rather promising, and they can have a significant impact in research, education, and communication of engineering science. The results can be used to fulfill the graduation requirements of some of the students of the work group. One important result of this program was the intercultural experience having an English-speaking person within the group. It was the group consensus this experience was one of the high points of the summer program.

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