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Local fractional variational iteration method for Fokker-Planck equation on a Cantor set

Método iterativo fraccionado variacional local para la ecuación Fokker-Planck en un conjunto de Cantor

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ABSTRACT

Recently the local fractional operators have started to be considered a useful tool to deal with fractal functions defined on Cantor sets. In this paper, we consider the Fokker-Planck equation on a Cantor set derived from the fractional complex transform method. Additionally, the solution obtained is considered by using the local fractional variational iteration method.

RESUMEN

Recientemente se ha empezado a considerar a los operadores locales fraccionados como una herramienta útil para lidiar con funciones fractales definidas en conjuntos de Cantor. En este artículo, consideramos la ecuación Fokker-Planck en un conjunto de Cantor derivado de un método fraccionado de transformación complejo. Así mismo, la solución obtenida se considera al usar el método iterativo fraccionado variacional local.

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Fokker-Planck equation; local fractional variational iteration method; fractional complex transform method.

Palabras clave:
Ecuación Fokker-Planck; método iterativo fraccionado variacional local; método fraccionado de transformación complejo.

INTRODUCTION

The fractional complex transform (He, Elagan & Li, 2012; Li, Zhu & He, 2012), derived from the modified Riemann-Liouville derivative, is an efficient tool to convert fractional differential equations to conventional differential equations. Recently, the fractional complex transform with local fractional derivatives (Hu, Baleanu & Yang, 2013; Yang, 2012a) was applied to switch conventional differential equations to differential equations on Cantor sets derived from local fractional calculus, based on fractal geometry (Aharony & Feder, 1990; Barnsley, 1998; Mandelbrot, 1983).

The traditional second-order Fokker-Planck equation (see references Risken, 1989; Sadighi, Ganji & Sabzehmeidani, 2007, and the references therein) has a large deviation from the dynamics of the Brownian motion that makes it inadequate for research purposes. The fractional Fokker-Planck equation with fractional coordinates and time derivatives suggested by Zaslavsky (1994) from fractional order calculus theory (Kilbas, Srivastava & Trujillo, 2006; Miller & Ross, 1993; Podlubny, 1999) has a smaller deviation from the dynamics of the Brownian, and was used to describe the anomalous behavior of the transport process through the fractal medium with the mean-square displacement (Chow & Liu, 2004; El-Wakil

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& Zahran, 2000; Henry, Langlands & Straka, 2010; Lenzi, Malacarne, Mendes & Pedron, 2003; Metzler, Barkai & Klafter, 1999; Metzler & Nonnenmacher, 2002; Sokolov, 2001; Tarasov, 2005). A solution to the fractional Fokker-Planck equation was investigated in the following ways: usingthe finite difference method (Chen, Liu, Zhuang & Anh, 2009), the Adomian decomposition method (Odibat & Momani, 2007; Tatari, Dehghan & Razzaghi, 2007), the variational iteration method (Odibat & Momani, 2007; Biazar, Gholamin & Hosseini, 2010), the homotopy perturbation method (Yıldırım, 2010) and others (Baleanu, Diethelm, Scalas & Trujillo, 2012).

Local fractional operators have local fractal behaviors (Yang, 2011a, 2011b, 2012b), such as the Kolwankar-Gangal local fractional derivative (Kolwankar & Gangal, 1998; Yang, 2011a, 2011b, 2012b), the Parvate-Gangal fractal derivative (Parvate & Gangal, 2005, 2009; Yang, 2011a, 2011b, 2012b), the Chen's fractal derivative (Chen, 2006; Chen, Sun, Zhang & Koroak, 2010; Yang, 2011a, 2011b, 2012b), the generalized fractal derivative (Chen *et al*, 2010), the Adda-Cresson local fractional derivative (Adda & Cresson, 2001; Yang, 2011a, 2011b, 2012b), the He's fractal derivative (He, 2011, 2012; He *et al.*, 2012; Li *et al.*, 2012; Yang, 2011a, 2011b, 2012b), the modified Riemann-Liouville derivative (Jumarie, 2007; Yang, 2011a, 2011b, 2012b), and the local fractional derivative structured in (Hu, Agarwal & Yang, 2012; Liao, Yang & Yan, 2013; Yang, 2011a, 2011b, 2012a, 2012b, 2012c; Yang & Baleanu, 2012; Yang, Baleanu & Zhong, 2013; Yang & Zhang, 2012).

It was applied to model problems for fractal mathematics and engineering on Cantor sets (Adda & Cresson, 2001; Chen, 2006; Chen *et al.*, 2010; He, 2011, 2012; He *et al.*, 2012; Hu *et al.*, 2012; Hu *et al.*, 2013; Jumarie, 2007; Kolwankar & Gangal, 1998; Li *et al.*, 2012; Liao *et al.*, 2013; Parvate & Gangal, 2005, 2009; Yang, 2011a, 2011b, 2012a, 2012b, 2012c; Yang & Baleanu, 2012; Yang *et al.*, 2013; Yang & Zhang, 2012).

Recently, the local fractional variational iteration method was proposed via local fractional calculus (Yang, 2012c; Yang & Baleanu, 2012; Yang & Zhang, 2012).

Our main purpose of the manuscript is to apply the fractional complex transform method to obtain the Fokker-Planck equations on a Cantor set and to investigate a solution to the linear Fokker-Planck equation on a Cantor set by using the local fractional variational iteration method. In this manuscript, Section 2 is devoted to the linear and nonlinear Fokker-Planck equations on a Cantor set using the local fractional complex transform method. The local fractional variational iteration method for the linear Fokker-Planck equation on a Cantor set is shown in Section 3. Finally, conclusions are given in Section 4.

Fokker-Planck equations on cantor sets

The Fokker–Planck equation is applied to describe the Brownian motion of particles. The conventional Fokker–Planck equation can be written as (Risken, 1989; Sadighi *et al.*, 2007)

$$\frac{\partial u}{\partial t}=\left[-\frac{\partial A(x)}{\partial x}+\frac{\partial^2 B(x)}{\partial x^2}\right]u, \tag{1}$$

or

$$\frac{\partial u}{\partial t}=\left[-\frac{\partial A(x,t)}{\partial x}+\frac{\partial^2 B(x,t)}{\partial x^2}\right]u, \tag{2}$$

subject to the initial condition

$$u(x,0)=f(x). \tag{3}$$

We generalized equation (2) to N variables $x_1, x_2, x_3, \cdots, x_N$ (Sadighi *et al.*, 2007)

$$\frac{\partial u}{\partial t}=\left[-\sum_{i=1}^{N}\frac{\partial A(x_1,x_2,x_3,\cdots,\mathrm{x}_N,t)}{\partial x_i}+\sum_{i,j=1}^{N}\frac{\partial^2 B(x_1,x_2,x_3,\cdots,x_N,t)}{\partial x_i\,\partial x_j}\right]u, \tag{4}$$

with the initial condition given by

$$u(x_1,x_2,x_3,\cdots,x_N,0)=f(x_1,x_2,x_3,\cdots,x_N). \tag{5}$$

A form of nonlinear Fokker–Planck equation was described as (Sadighi, *et al.*, 2007)

$$\frac{\partial u}{\partial t}=\left[-\frac{\partial^\alpha A(x,u,t)}{\partial x}+\frac{\partial^2 B(x,u,t)}{\partial x^2}\right]u, \tag{6}$$

and a more general form of the nonlinear Fokker–Planck equation was given as (Sadighi *et al.*, 2007)

$$\frac{\partial u}{\partial t}=\left[-\sum_{i=1}^{N}\frac{\partial B(x_1,x_2,x_3,\cdots,x_N,u,t)}{\partial x_i}+\sum_{i,j=1}^{N}\frac{\partial^2 B(x_1,x_2,x_3,\cdots,x_N,u,t)}{\partial x_i\partial x_j}\right]u, \tag{7}$$

where the initial condition is

$$u\left(x_1,x_2,x_3,\cdots,x_N,0\right)=f\left(x_1,x_2,x_3,\cdots,x_N\right). \quad (8)$$

By fractional complex transform method (Hu *et al.*, 2013; Yang, 2012a),

$$\begin{cases} T=\dfrac{t^\alpha}{\Gamma\left(1+\alpha\right)} \\ X=\dfrac{x^\alpha}{\Gamma\left(1+\alpha\right)} \end{cases}, \quad (9)$$

Equation (2) is converted as

$$\frac{\partial^\alpha u}{\partial t^\alpha}=\left[-\frac{\partial^\alpha A\left(x,t\right)}{\partial x^\alpha}+\frac{\partial^{2\alpha} B\left(x,t\right)}{\partial x^{2\alpha}}\right]u, \quad (10)$$

subject to the fractal initial condition

$$u\left(x,0\right)=f\left(x\right), \quad (11)$$

where the local fractional differential operator is written as

$$u^{(\alpha)}\left(x_0,y\right)=\frac{\partial^\alpha u\left(x,y\right)}{\partial x^\alpha}\Big|_{x=x_0}=\lim_{x\to x_0}\frac{\Delta^\alpha\left(u\left(x,y\right)-u\left(x_0,y\right)\right)}{\left(x-x_0\right)^\alpha}, \quad (12)$$

with

$$\Delta^\alpha\left(u\left(x,y\right)-u\left(x_0,y\right)\right)\cong\Gamma\left(1+\alpha\right)\Delta\left(u\left(x,y\right)-u\left(x_0,y\right)\right).$$

In a similar manner, a generalization of equation (4) takes the form

$$\frac{\partial^\alpha u}{\partial t^\alpha}=\left[-\sum_{i=1}^{N}\frac{\partial^\alpha A\left(x_1,x_2,x_3,\cdots,x_N,t\right)}{\partial x_i^\alpha}+\sum_{i,j=1}^{N}\frac{\partial^{2\alpha} B\left(x_1,x_2,x_3,\cdots,x_N,t\right)}{\partial x_i^\alpha\,\partial x_j^\alpha}\right]u, \quad (13)$$

with the fractal initial condition given by

$$u\left(x_1,x_2,x_3,\cdots,x_N,0\right)=f\left(x_1,x_2,x_3,\cdots,x_N\right). \quad (14)$$

and a generalization of the nonlinear Fokker–Planck equation on a Cantor set is addressed as

$$\frac{\partial^\alpha u}{\partial t^\alpha}=P_\alpha u, \quad (15)$$

where the nonlinear local fractional Fokker–Planck operator is presented as

$$P_\alpha=-\frac{\partial^\alpha A\left(x,u,t\right)}{\partial x^\alpha}+\frac{\partial^{2\alpha} B\left(x,u,t\right)}{\partial x^{2\alpha}}. \quad (16)$$

A generalization of nonlinear Fokker–Planck equation on a Cantor set to N variables $x_1,x_2,x_3,\cdots,x_N$ has the form

$$\frac{\partial^\alpha u}{\partial t^\alpha}=P_{\alpha,\mathrm{N}}u. \quad (17)$$

with the nonlinear local fractional Fokker–Planck operators

$$P_{\alpha,N}=\left[-\sum_{i=1}^{N}\frac{\partial^\alpha A\left(x_1,x_2,x_3,\cdots,x_N,t\right)}{\partial x_i^\alpha}+\sum_{i,j=1}^{N}\frac{\partial^{2\alpha} B\left(x_1,x_2,x_3,\cdots,x_N,t\right)}{\partial x_i^\alpha\,\partial x_j^\alpha}\right]t, \quad (18)$$

which is satisfied with the local fractional continuous condition,

$$u\left(x_1,x_2,x_3,\cdots,x_N,t_1\right)-u\left(x_1,x_2,x_3,\cdots,x_N,t_0\right)\Big|\le\varepsilon^\alpha, \quad (19)$$

We derive the special forms of equations (10, 17) as follows:

$$\frac{\partial^\alpha u}{\partial t^\alpha}=-\frac{\partial^\alpha u}{\partial x^\alpha}+\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}}, \quad (20)$$

and

$$\frac{\partial^\alpha u}{\partial t^\alpha}=-R\left(u,x\right)\frac{\partial^\alpha u}{\partial x^\alpha}+S\left(u,x\right)\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}}, \quad (21)$$

where both $R\left(u,x\right)$ and $R\left(u,x\right)$ are Polynomials of u and x in the special case of equations (10, 17). Equation (20) is the so-called linear Fokker–Planck equation on a Cantor set while equation (21) is a nonlinear one in one-dimensional Cantorian time spaces. We present the fracttal condition as

$$u\left(x,0\right)=f\left(x\right), \quad (22)$$

which leads to

$$\left|f\left(x\right)-f\left(x_0\right)\right|\le\varepsilon^\alpha. \quad (23)$$

In view of equation (23), we obtain (Hu *et al.*, 2012)

$$\rho^\alpha\left|x_1-x_2\right|^\alpha\le\left|f\left(x_1\right)-f\left(x_2\right)\right|\le\tau^\alpha\left|x_1-x_2\right|^\alpha, \quad (24)$$

which yields

$$\rho^{\alpha} H^{\alpha}\left(F\right) \leq H^{\alpha}\left(f\left(F\right)\right) \leq \tau^{\alpha} H^{\alpha}\left(F\right), \tag{25}$$

where $H^{s}\left(\bullet\right)$ is defined as the Hausdorff measure.

From equation (25) it follows that α is a fractal dimension of the set F and family of the function defined on set F *or* $f\left(F\right)$. It is found that Fokker–Planck equations on a Cantor set differ from the results (Henry *et al.*, 2010; Metzler *et al.*, 1999; Metzler & Nonnenmacher, 2002).

The local fractional variational iteration method for the linear Fokker-Planck equation on a Cantor set

The local fractional variational iteration method via local fractional calculus was proposed in (Yang, 2012c; Yang & Baleanu, 2012; Yang & Zhang, 2012). We read a correction local fractional functional as follows:

$$u_{n+1}\left(t\right) = u_{n}\left(t\right) + {}_{0}I_{t}^{(\alpha)}\left\{\xi^{\alpha}\left[L_{\alpha}u_{n}\left(s\right) + N_{\alpha}\tilde{u}_{n}\left(s\right) - g\left(s\right)\right]\right\}, \tag{26}$$

where the local fractional integral operator is defined through

$$ {}_{t_0}I_{t}^{(\alpha)} f\left(t\right) = \frac{1}{\Gamma\left(1+\alpha\right)}\int_{a}^{b} f\left(t\right)\left(dt\right)^{\alpha} = \frac{1}{\Gamma\left(1+\alpha\right)}\lim_{\Delta t\to 0}\sum_{j=0}^{j=N-1} f\left(t_{j}\right)\left(\Delta t_{j}\right)^{\alpha},$$

with a partition of the interval $[a,b]$ given by $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max\left\{\Delta t_1, \Delta t_2, \Delta t_j, \ldots\right\}$, $\left[t_j, t_{j+1}\right]$, $j = 0,\ldots, N-1$, $t_0 = a$, $t_N = b$, where the local fractional differential equation is

$$L_{\alpha}u\left(s\right) + N_{\alpha}u\left(s\right) = g\left(s\right), \tag{27}$$

with a linear local fractional differential operator L_{α}, a nonlinear local fractional operator N_{α} and an inhomogeneous term of non-differential function $g(s)$ and where $\delta^{\alpha}\tilde{u}_{n}$ is considered as a restricted local fractional variation and ξ^{α} is a fractal Lagrange multiplier. i.e. $\delta^{\alpha}\tilde{u}_{n} = 0$ (Yang, 2012b). Consequently, the solution is

$$u = \lim_{n\to\infty} u_{n}. \tag{28}$$

We take into account equation (20) with a fractal initial condition given by

$$u\left(x,0\right) = f\left(x\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)}. \tag{29}$$

In view of equatio (26), we determine the iteration formula as follows:

$$\delta^{\alpha}u_{n+1}\left(t\right) = \delta^{\alpha}u_{n}\left(t\right) + \delta^{\alpha}\,{}_{0}I_{t}^{(\alpha)}\left\{\xi^{\alpha}\left[\frac{\partial^{\alpha}u}{\partial t^{\alpha}} + \frac{\partial^{\alpha}u}{\partial x^{\alpha}} - \frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}\right]\right\}, \tag{30}$$

where $\delta^{\alpha}\tilde{u}_{n}$ is considered as a restricted local fractional variation.

The stationary conditions of equation (30) read as

$$\left(\xi^{\alpha}\left(\tau\right)\right)^{(\alpha)} = 0, \tag{31}$$

$$1 + \xi^{\alpha}\left(\tau\right)\Big|_{\tau=t} = 0. \tag{32}$$

Hence, the fractal Lagrange multiplier is simply identified as

$$\xi^{\alpha}\left(\tau\right) = -1. \tag{33}$$

Submitting equation (33) to equation (30), there is an iteration formula

$$u_{n+1}\left(t\right) = u_{n}\left(t\right) + {}_{0}I_{t}^{(\alpha)}\left\{-\left[\frac{\partial^{\alpha}u_{n}}{\partial t^{\alpha}} + \frac{\partial^{\alpha}u_{n}}{\partial x^{\alpha}} - \frac{\partial^{2\alpha}u_{n}}{\partial x^{2\alpha}}\right]\right\}. \tag{34}$$

Starting with a fractal initial condition, $u\left(x,0\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)}$, one reaches at the iteration formula:

$$u_{0}\left(x,t\right) = u\left(x,0\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)}, \tag{35}$$

$$u_{1}\left(x,t\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)} + \frac{t^{\alpha}}{\Gamma\left(1+\alpha\right)}, \tag{36}$$

$$u_{2}\left(x,t\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)} + \frac{t^{\alpha}}{\Gamma\left(1+\alpha\right)}. \tag{37}$$

Continuing to compute them in this manner, for $n > 1$ we can give

$$u_{n+1}\left(x,t\right) = u_{n}\left(x,t\right). \tag{38}$$

Following equation (28), an exact solution to equation (20) can be hence obtain in the form of

$$u\left(x,t\right) = \frac{x^{\alpha}}{\Gamma\left(1+\alpha\right)} + \frac{t^{\alpha}}{\Gamma\left(1+\alpha\right)}. \tag{39}$$

CONCLUSIONS

In this paper, using the fractional complex transform method via a local fractional chain rule, we point out the linear and nonlinear Fokker-Planck equations on a Cantor set. The obtained equations are structured in a fractal time space, whose element of fractal arc length squared can be seen (Yang, 2012b; Yang *et al.*, 2013). The exact solution of the linear Fokker-Planck equation on a Cantor set is obtained by the local fractional variational iteration method, which is an efficient tool for scientists and engineers to obtain solutions to the differential equations using local fractional derivatives.

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